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THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED
WITH THE CO-OPERATION OF THE

AMERICAN PHYSICAL SOCIETY

BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

CLX

JULY, 1909

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THE PHYSICAL REVIEW.

SOME ELECTRICAL PROPERTIES OF SELENIUM. I.

By Louise S. McDowell.

IN the thirty-five years since the discovery was made that the resistance of selenium diminishes under the action of light, many investigators have studied the phenomenon and several theories have been developed to account for the action. Little attention, however, has been paid to the form of the recovery curves, *i. e.*, the curves which show the rate of return to the normal dark resistance after exposure to the light has ceased, although it has been frequently noted that the lowering of resistance under the action of light is rapid, while the return to the dark resistance is much slower, occupying from a few minutes to hours or even days according to the intensity and duration of the excitation.

In the hope that a study of the form of the recovery curves as affected by a variation in the initial conditions might throw some light upon the general problem, the present experiments were undertaken at the suggestion of Professor Merritt, and to him and to Professor Nichols the author wishes to express her grateful appreciation for their unfailing interest and inspiring suggestions. The variations in initial conditions studied were of three kinds: (1) a variation in the wave-length of the exciting light; (2) a variation in the intensity of excitation for constant time of exposure; (3) a variation in the duration of exposure at constant intensity. Since the excitation as well as the recovery is a process requiring time, a further study was made of the effect upon the saturation

curves of (1) change in the intensity of the exciting light; (2) change in the wave-length of the exciting light.

The first experiments were concerned with the variation in the form of the recovery curve produced by a variation in the wave-length and in the intensity of the exciting light, the time of exposure remaining constant. A few earlier investigators have shown such curves. Athanasiadis,[1] for example, allowed the selenium cell to be excited a long time and then suddenly brought into darkness. The time of recovery for a cell thus excited to saturation, he finds to be approximately fifty or sixty hours. The curves are plotted as time resistance curves and show in a general way the initial rapid recovery gradually diminishing in rapidity as the dark resistance is approached. Ruhmer[2] studied the excitation and recovery of five cells which he divides into two kinds, "hard" and "soft." Soft cells were those annealed at a temperature of 200°; hard cells those annealed at a lower temperature and cooled more quickly. The soft cells he found to be relatively more sensitive to weak excitations, while the hard cells showed a more rapid recovery than the soft. The curves as plotted show the relation of resistance to time, both for five minute excitation and for the ensuing recovery. The data when replotted with the reciprocal square roots of the changes in conductivity as the ordinates, give curves similar in form to those obtained by the writer.

The general method of the experiments carried on by the writer was to measure the dark resistance of the cell, then expose it for a definite number of seconds to the light, note the resistance at the instant when the light was shut off and at frequent intervals during the return to the normal dark resistance. The cells used were both of the Bidwell[3] and the Ruhmer types,[4] while for the excitation there were used the light of the arc lamp dispersed by a flint-glass prism and the light of a 16-c.p. incandescent lamp screened by

[1] G. Athanasiadis, Annalen der Physik, Vol. 25, p. 1.

[2] Ernst Ruhmer, Phys. Zs., Vol. 3, p. 468.

[3] In the Bidwell cell the electrodes are two fine platinum or copper wires perhaps a meter in length, wound in alternate notches upon a rectangular plate of mica or slate, at distances apart varying from .06 to .1 cm. The thin coating of selenium is melted upon one side and the cell sensitized by prolonged heating.

[4] In the Ruhmer type the wires are wound upon a porcelain cylinder enclosed in a glass vacuum tube.

red and green glasses. As a current source a single storage cell was used throughout the experiments. The resistance was measured by means of a Wheatstone bridge of which the cell formed one arm. The bridge was first balanced for a known resistance approximately equal to that of the cell and calibrated by substituting known resistances in place of the cell and noting the deflections produced upon the graduated scale by a beam of light reflected from the galvanometer mirror. A curve was plotted with scale deflections as ordinates and the corresponding resistances above and below equilibrium resistance as abscissæ. From this curve the resistance of the cell for any observed deflection could be determined. The method, while less accurate than the zero method, possessed the obvious advantage of rapidity, thus making possible the obtaining of any desired number of points upon the recovery curves.

In the early experiments with the Bidwell cell the dark resistance of the cell was found to be far from steady, especially on dark days. The probable sources of error seemed to be inconstancy of galvanometer, variation in temperature of the cell, and the presence of a film of moisture. The first of these errors was found upon testing the galvanometer with a standard cell and known resistance to be negligible. The second and especially the third were the more serious. To obviate these difficulties the cell was placed in a water-bath with a glass window and with the cell there was included a calcium chloride tube. With these precautions the dark resistance remained comparatively steady. After each run the cell was allowed to remain in the dark, on open circuit, approximately twenty-four hours although no attempt was made to bring the initial resistance exactly to the same point before each experiment. To avoid errors due to a change in the strength of the current the galvanometer was frequently tested and if necessary recalibrated. As a final precaution, to prevent the admission of stray light to the cell, the dispersed light was allowed to pass through a light-tight box against the farther end of which the cell was placed. A metal shutter protected the face of the cell when not exposed. (See Fig. 1.)

In the first set of observations taken the exciting light was a 16-c.p. incandescent lamp, the light from which passed through a

small circular opening in a tin screen and fell upon the colored glass window of a light-tight box. In this case a 10 cm. × 10 cm. water-cell was placed in the box to prevent any effect due to change in temperature, and the intensity of the light was varied by changing the size of the opening. The cell used was of the Bidwell type with a dark resistance of approximately 60,000 ohms. Observations were made for light passing through red glass; through a

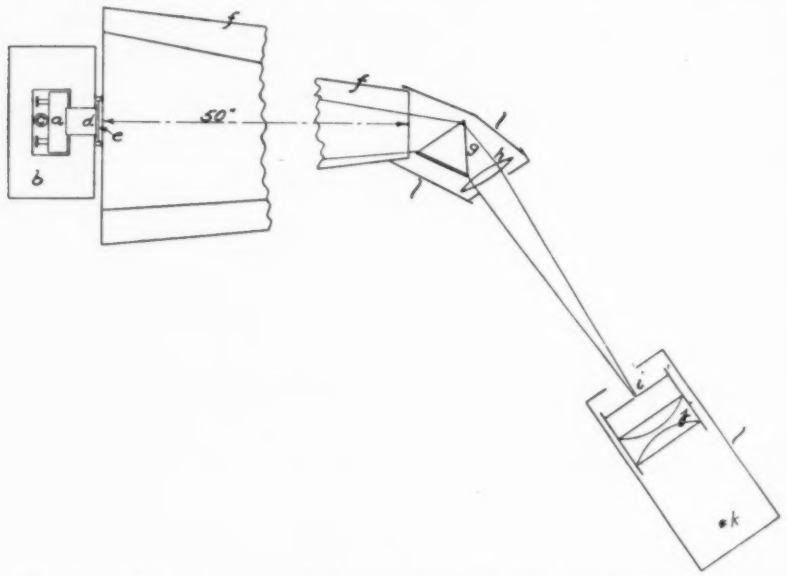


Fig. 1. *a*, Cell; *b*, tin water-bath; *c*, calcium chloride tube; *d*, glass window; *e*, metal shutter; *f*, box to prevent admission of stray light; *g*, flint glass prism; *h*, focusing lens; *i*, adjustable slit; *j*, condensing lenses; *k*, arc; *l*, screens.

combination of two glasses, green and blue, since neither alone was found sufficient entirely to shut out the red rays; and finally through thin rubber. The uniform time of excitation was 20 seconds.

In the remaining experiments the arc lamp was used for excitation as described above. In the second of the two sets there was used a cell of the Bidwell type manufactured by Max Kohl, with a dark resistance of approximately 500,000 ohms at room temperatures, and exceedingly sensitive. The third set of data was obtained with a cell of the Ruhmer type also made by Max Kohl, with a dark

resistance of about 45,000 ohms. The cell was exhausted by a Gaede mercury pump and freed from moisture by repeated exhaustion and admission of dry air. It was then placed in a light-tight wooden box closed on one side by a brass shutter so that one half the selenium surface was exposed to the exciting light. In both these sets the time of excitation was 5 seconds. Readings were taken for light of three wave-lengths, blue-violet, red and infra-red. In the case of the infra-red the possibility of stray light was avoided by passing the dispersed light through a thin sheet of rubber. For convenience the essential facts concerning each set of observations are given below in tabular form.

Set.	No. of Cell.	Type.	Approximate Dark Resistance.	Source of Light.	Time of Excitation.	Figs.
I.	I.	Bidwell	60,000	16 c.p. lamp	20 sec.	4– 7
II.	*A*	"	500,000	Carbon arc	5 "	8–11
III.	*B*	Ruhmer	45,000	" "	5 "	12–15
IV.	*C*	Bidwell	9,000	" "	5 "	16–19

Results of Observations.

As was to be expected the time of recovery was longer when the intensity was greater, but the form of the curve was found to be essentially the same for each color and for each intensity. The curves were plotted with the reciprocal square roots of the change in conductivity, $1/\sqrt{C - C_0}$, as ordinates and the corresponding times as abscissæ. The form of the curve so plotted is similar to the curves obtained for the decay of phosphorescence in Sidot blende by Nichols and Merritt, whose results suggested this method of plotting. The curve consists of two parts nearly straight, but of decidedly different slope, connected by a gradual bend. Usually neither part is exactly straight but bends somewhat towards the axis of abscissæ. Fig. 2 shows two curves, however, in which the lines are approximately straight. These curves suggest that after prolonged rest and at absolutely constant temperature the curves might be of that form. The first of these curves was obtained with Cell I., which had been kept in darkness, on open circuit, five days, and was then excited three minutes by red light and the recovery studied for three hours. The cell used requires thirty minutes for

even approximate saturation. The second of the two curves was obtained by a very brief iron spark excitation of possibly not more than a single spark. In this case the straight line form was assumed

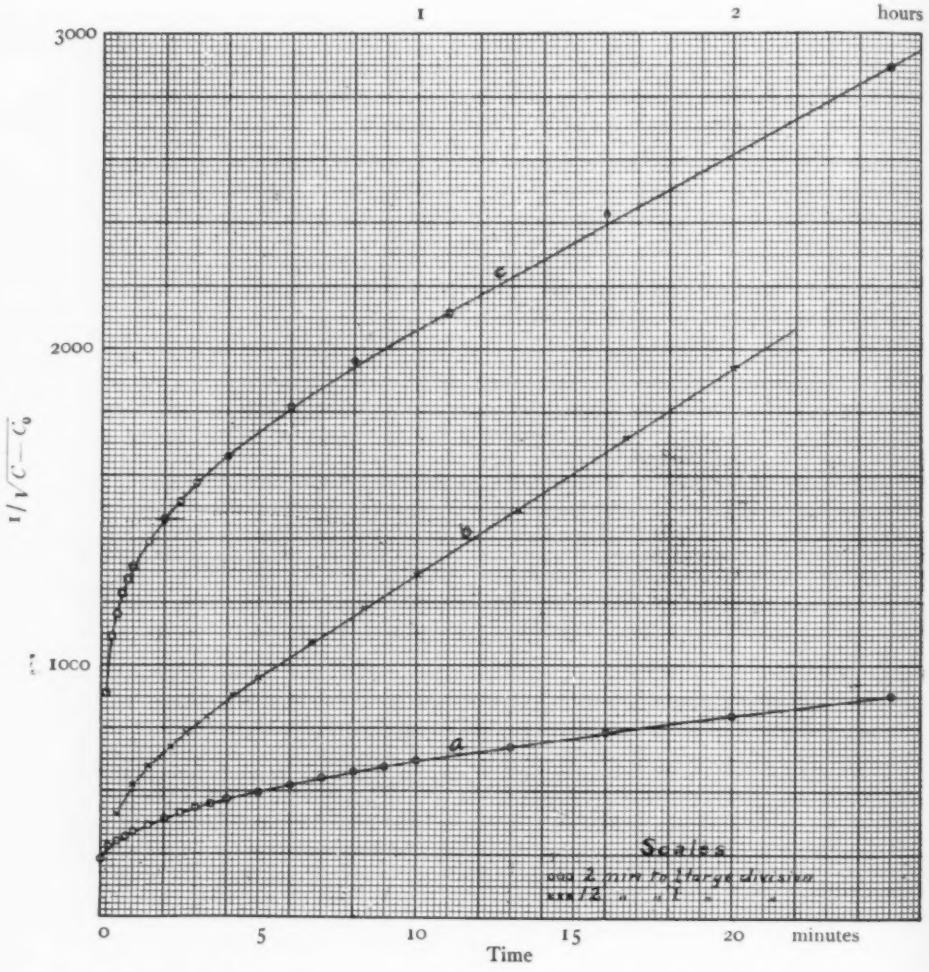


Fig. 2. Curves to show general type of recovery after prolonged rest. Cell I. *a*, *b*, Three minutes excitation by red light after five days in darkness; *c*, brief iron spark excitation.

after about ten minutes, whereas with the longer excitation there was a gradual bending during the first hour of decay.

The relation of the plotted curves to the actual observations is

shown in Fig. 3, where the same data are plotted in three different ways. In curve *a* the ordinates are the observed resistances and the abscissæ the times. This curve approaches asymptotically the line of dark resistance *b*. In *c* the ordinates are the conductivities. This curve in like manner approaches the line of constant dark conductivity *d*. In *e* the data are plotted by the usual method. The

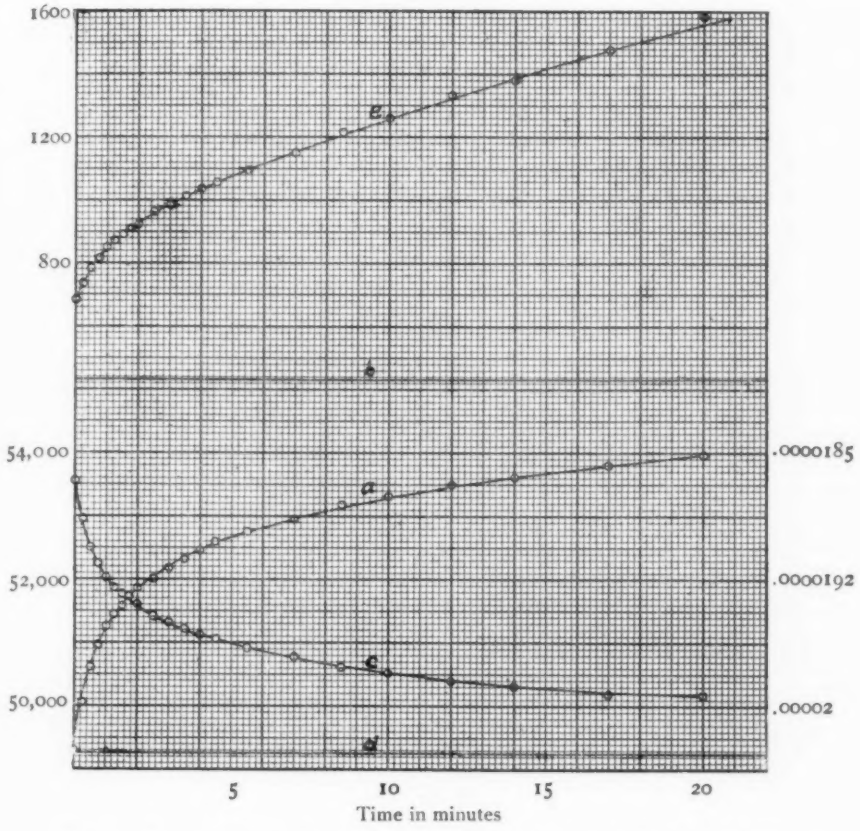


Fig. 3. Curves showing the relation of the plotted curves to the observations. *a*, Observed resistances as ordinates; *b*, line of constant dark resistance; *c*, conductivities as ordinates; *d*, line of constant dark conductivity; *e*, reciprocal square roots of the change in conductivity as ordinates.

quantities used as ordinates are the reciprocal square roots of the differences between the ordinates *c* and *d* ($1/\sqrt{C-C_0}$). As the values of $C-C_0$ become small, *i. e.*, as the resistance approaches

the initial dark resistance, evidently slight errors in either C or C_0 will produce relatively large errors in the corresponding values of the ordinates of the derived curve. Hence slight variations in the form of the latter part of the curve are of less importance.

In order accurately to compare the form of the curves for the

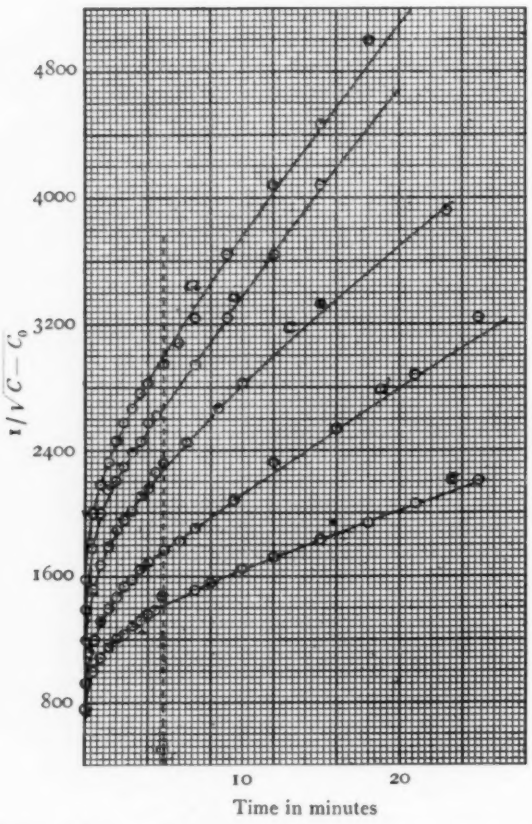


Fig. 4. Variation in the rate of recovery with intensity of excitation. Cell I. Twenty seconds excitation by green light. Initial dark resistances: *a*, 56,200; *b*, 51,090; *c*, 52,520; *d*, 52,660; *e*, 55,290.

different colors, it was necessary to obtain curves for each color which should start from the same initial value of the reciprocal, $1/\sqrt{C-C_0}$. To accomplish the reduction from the curves plotted from the observed data to these comparison curves the following

method was used. Through the original curves for any one color lines were drawn parallel to the axis of ordinates through different points on the axis of abscissæ. (See dotted line, Fig. 4.) Secondary curves were then plotted by taking the points of intersection of a single line with the various curves as the ordinates of points

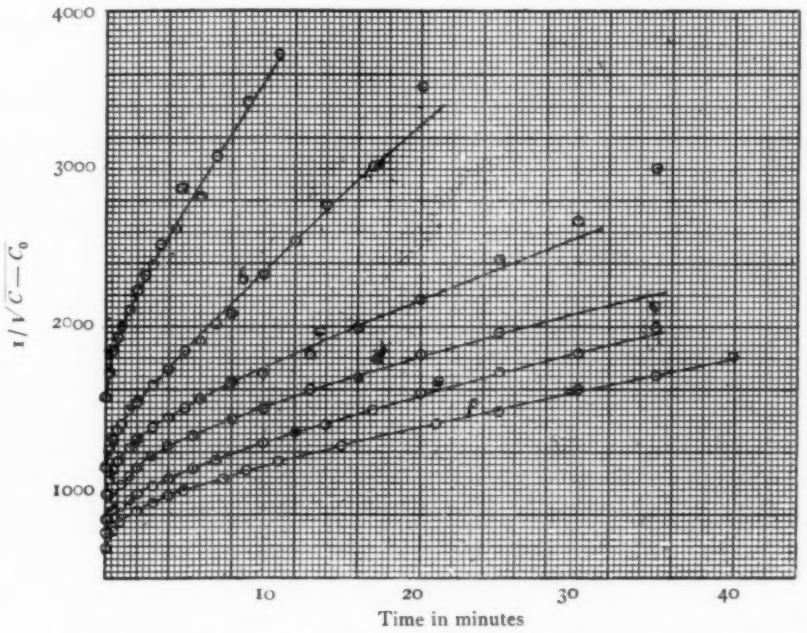


Fig. 5. Variation in the rate of recovery with intensity of excitation. Cell I. Twenty seconds excitation by red light. Initial dark resistances; *a*, 53,920; *b*, 51,040; *c*, 51,800; *d*, 52,070; *e*, 55,160; *f*, 54,010.

the abscissæ of which were the ordinates of the initial points of the curves so intersected. These points served to determine a curve which gave the value of the ordinate, $1/\sqrt{C-C_0}$, at the particular time chosen, for any desired value of the ordinate of the initial point. For example, the curve for 30 seconds for red light gave the value of $1/\sqrt{C-C_0}$ for each of the curves of Fig. 4, 30 seconds after recovery began. By constructing a series of secondary curves for different times sufficient points were obtained to plot the curves of Fig. 7.

The sets of color curves for the four cells used are shown in Figs.

4–19. From these curves it will be noted that in each case the recovery after infra-red excitation is much slower than after red or green or violet. The difference between the two colors is not so

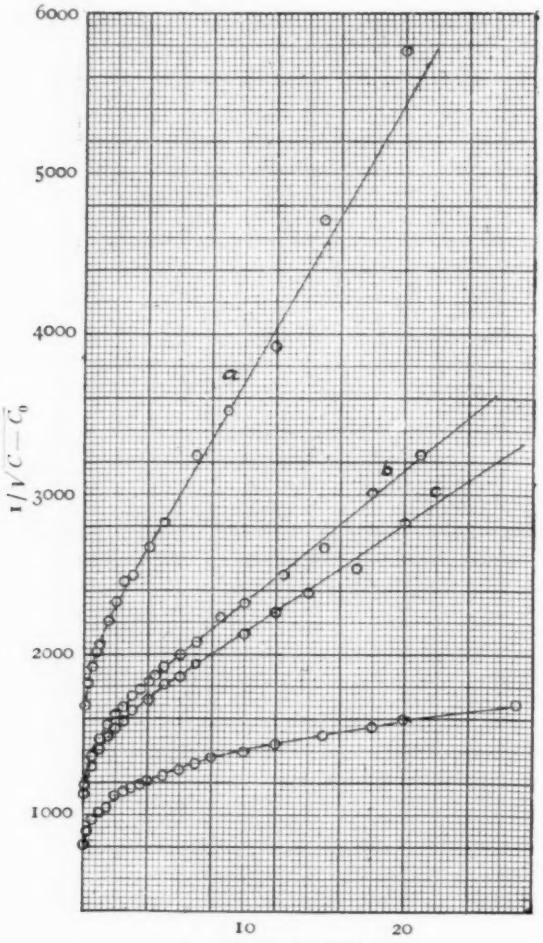


Fig. 6. Variation in the rate of recovery with intensity of excitation. Cell I. Twenty seconds excitation by infra-red light. Initial dark resistances: *a*, 53,920; *b*, 54,050; *c*, 52,210; *d*, 54,290.

marked. Cells A and C of the same type and make, but of widely differing sensibility, show the most rapid recovery for the red light while Cells I. and B show the quickest recovery for the shortest wave-length.

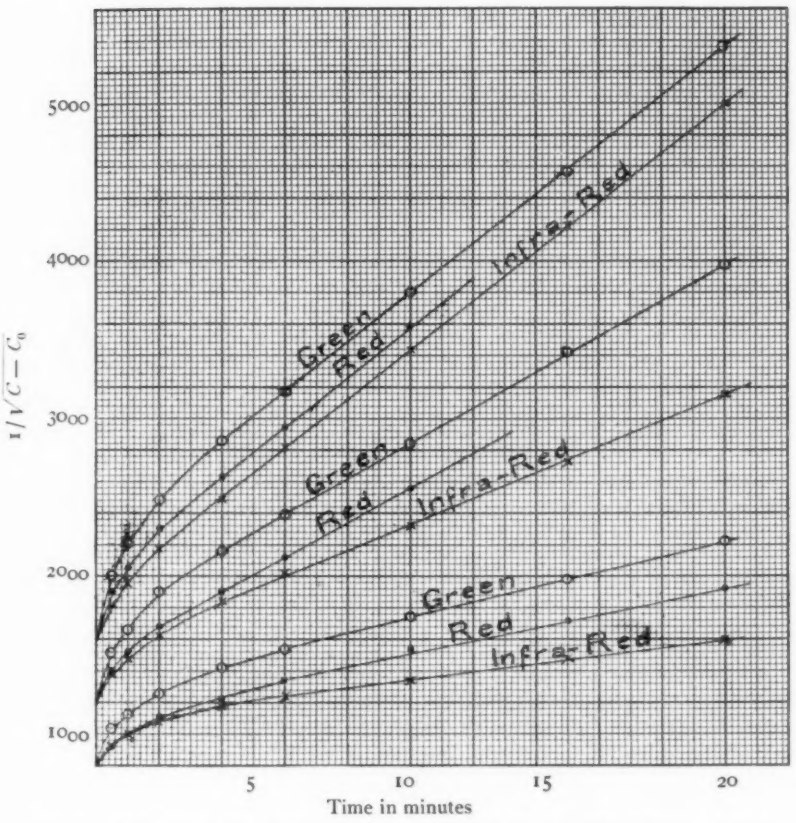


Fig. 7. Variation in rate of recovery with change in wave-length of light. Cell I.

Variation with Time of Excitation.

To determine the effect of time of excitation upon the form of the recovery curve, another set of observations was taken with the same apparatus as in the first set for study of intensity. For excitation both red and green lights were used. Curves were taken for times ranging from ten seconds to a time which produced saturation. The cells chosen were Cells I. and II., a home-made cell of the Bidwell type with a dark resistance of approximately 60,000 ohms. Both cells were extremely slow in responding to excitation and in returning to the dark resistance. Figs. 20–22 show the results plotted in the usual manner. The effect of longer time is similar to the

effect of increased intensity in making the recovery slower. The curves for Cell II., especially, show for very slight differences in the initial values a rapid decrease in the slope of the curve as the time of excitation is increased. It will be noted also that all the long-

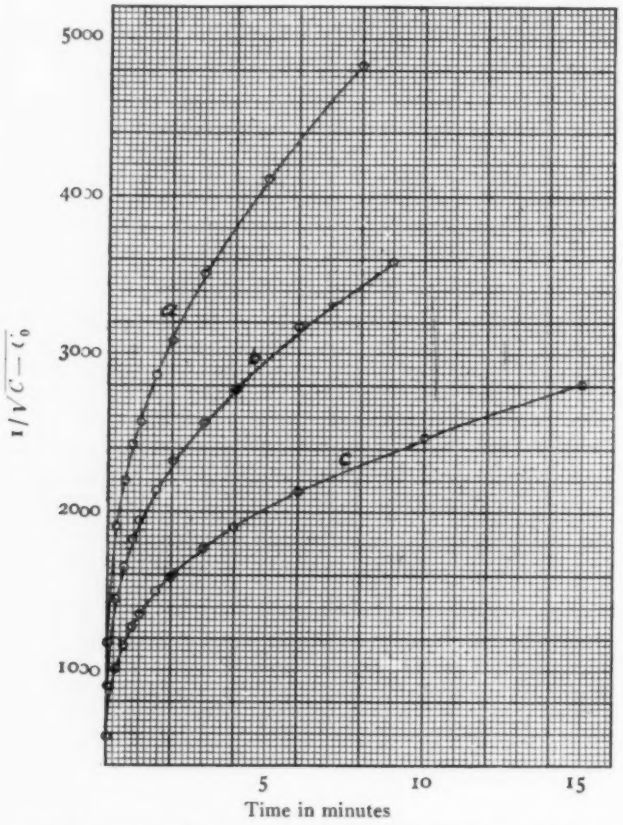


Fig. 8. Variation in rate of recovery with intensity of excitation. Cell A. Five seconds excitation by violet light. Initial dark resistances: *a*, 435,000; *b*, 459,000; *c*, 475,000.

time curves show a decided tendency to bend towards the axis of abscissæ.

SATURATION CURVES.

The inertia of selenium, *i. e.*, the gradual change in resistance produced by continued lighting, has been frequently noted. Marc[1]

[1] Zs. für Anorgan. Chemie, Vol. 37, p. 459.

says that there is no inertia of selenium itself, but the appearance is due to the fact that the action of light is not limited to the exposed surface but spreads over the entire cell and presses into the interior. In the following study of the relation of the time of saturation to the intensity and wave-length of the exciting light, two cells were used: Cell I., which had shown the most rapid recovery for green

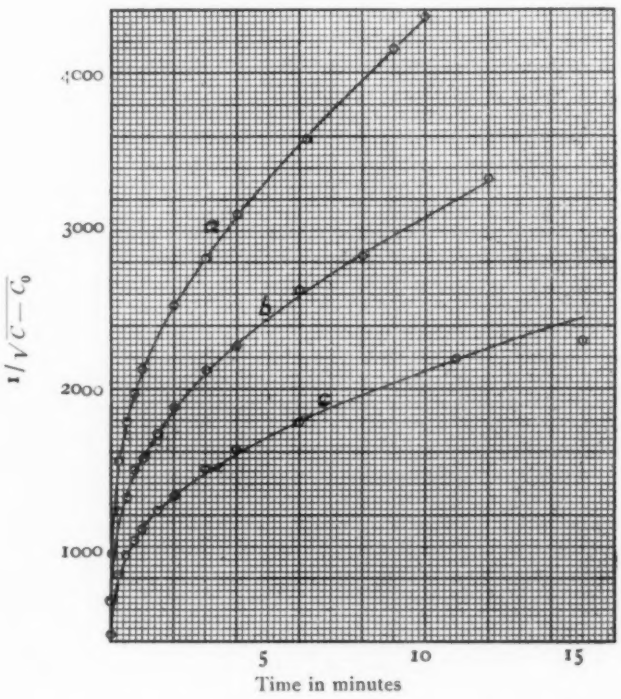


Fig. 9. Variation in rate of recovery with intensity of excitation. Cell A. Five seconds excitation by red light. Initial dark resistances: *a*, 461,000; *b*, 401,000; *c*, 440,000.

light and Cell A, which had shown the most rapid recovery for red light. The apparatus and the precautions taken were essentially the same as in the first set of color observations with two exceptions: the incandescent lamp was replaced by an acetylene flame; and for the first experiments the colored glass screen was omitted. The source of light was placed at distances of twenty-five, fifty, and one hundred centimeters and a saturation curve taken for each

distance. In Fig. 23 three resulting curves for white light excitation are plotted with the changes in conductivity ($C - C_0$) as ordinates and the times as abscissæ. The curves will be seen to rise rapidly at first and then bend until they become approximately parallel to the axis of abscissæ as saturation is approached. While complete saturation was not attained in four hours, 80 per cent. of the change took place in the first five minutes of excitation. In Table

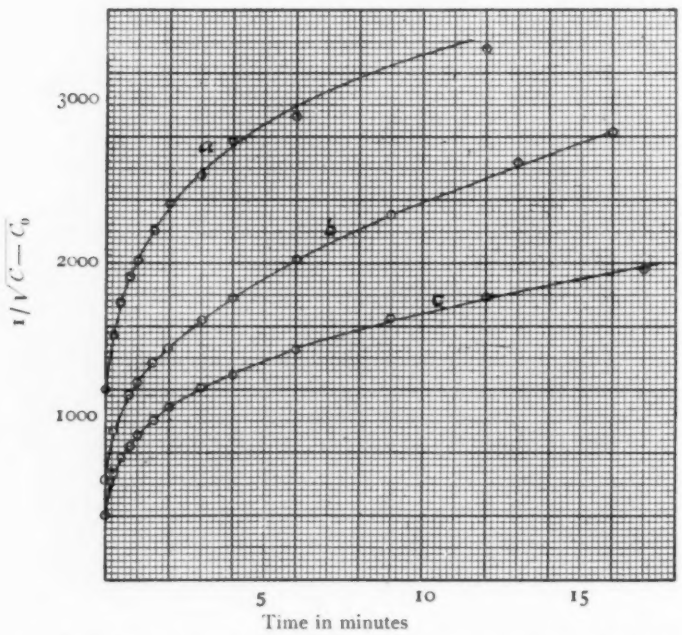


Fig. 10. Variation in rate of recovery with intensity of excitation. Cell A. Five seconds excitation by infra-red light. Initial dark resistances: *a*, 496,000; *b*, 488,000; *c*, 411,000.

I. are given the ratios of the changes in conductivity at different times, *e. g.*, the change in 45 seconds to the change in 4 hours. The results indicate that the change takes place more rapidly at first in the case of the more intense light but that the rate of excitation also diminishes more rapidly with intense light so that within five minutes the percentage changes have become very nearly equal.

The next attempt was to trace the dependence of time of saturation upon the color of the exciting light. For this purpose the

TABLE I.

Values of the Ratios, $(C_1 - C_0)/(C_2 - C_0)$, *for Different Times during Excitation.* (*Cell I.*)

	Curve I. (100 cm.)	Curve II. (50 cm.)	Curve III. (25 cm.)
45 sec. / 4 hrs.	.515	.663	.666
5 min. / 4 hrs.	.773	.848	.779
30 min. / 4 hrs.	.871	.933	.882
1 hr. / 4 hrs.	.927	.960	.928
1 min. / 1 hr.	.614	.734	.740
5 min. / 1 hr.	.834	.884	.840
30 min. / 1 hr.	.964	.974	.950

The ratios of the final resistance to the initial resistance were: 100 cm., 57.4 per cent.; 50 cm., 51.6 per cent.; 25 cm., 32.8 per cent.

colored glass screens were replaced in the apparatus. In Table II. are given for Cell I. the ratios for red and green light compared with similar ratios for white light of varying intensity.

It will be noted that although the final percentage change due to

TABLE II.

Values of the Ratios, $(C_1 - C_0)/(C_2 - C_0)$, *for Different Colors.* (*Cell I.*)

	Green.	Red.	White.	
			(100 cm.)	(50 cm.)
30 sec. / 45 min.	.310	.306	.481	.631
1 min. / 45 min.	.401	.459	.624	.745
5 min. / 45 min.	.741	.695	.846	.896
10 min. / 45 min.	.857	.802	.922	.935
30 min. / 45 min.	.979	.967	.976	.986

The ratios of the resistance attained in a 45 minute excitation to the initial resistance were: green, 76.4 per cent.; red, 70.5 per cent.; white, 100 cm., 59.7 per cent.; 50 cm., 54.6 per cent.

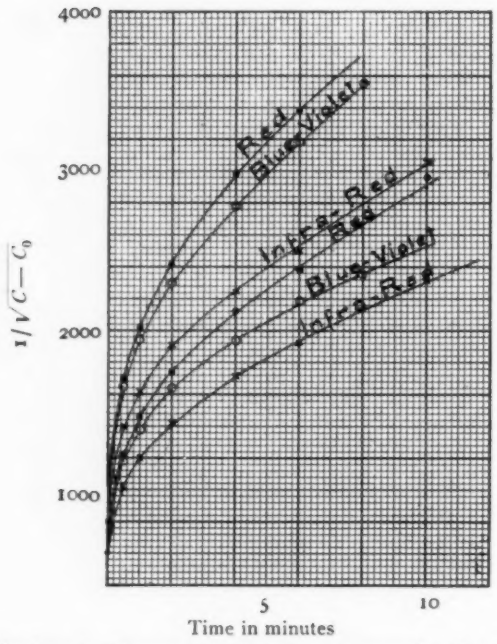


Fig. 11. Variation in rate of recovery with change in wave-length of light. Cell A.

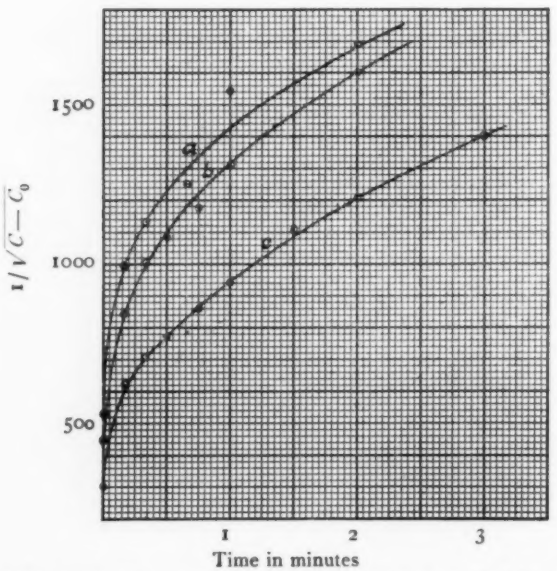


Fig. 12. Variation in rate of recovery with intensity of excitation. Cell B. Five seconds excitation by violet light. Initial dark resistances: *a*, 37,900; *b*, 39,700; *c*, 34,600.

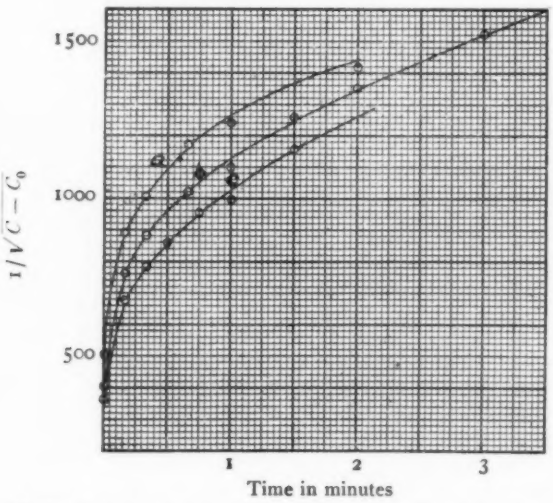


Fig. 13. Variation in rate of recovery with intensity of excitation. Cell B. Five seconds excitation by red light. Initial dark resistances: *a*, 37,500; *b*, 36,000; *c*, 35,100.

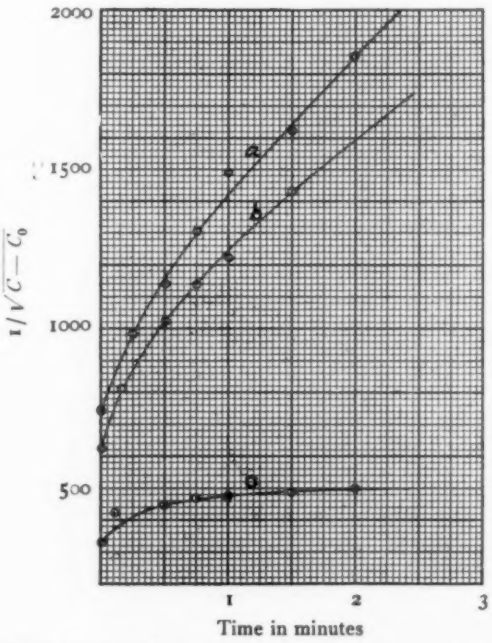


Fig. 14. Variation in rate of recovery with intensity of excitation. Cell B. Five seconds excitation by infra-red light. Initial dark resistances: *a*, 34,700; *b*, 33,750; *c*, 40,700.

the red was greater than that due to the green, the rate of saturation in the case of the green was greater than in the case of the red. Had

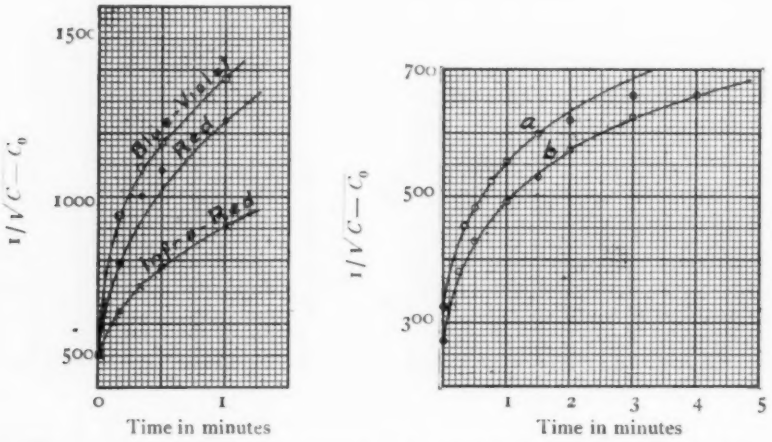


Fig. 15. Variation in rate of recovery with change in wave-length of light. Cell B.

Fig. 16. Variation in rate of recovery with intensity of excitation. Cell C. Five seconds excitation by violet light. Initial dark resistances: *a*, 8,595; *b*, 8,505.

the intensities been so chosen as to produce the same final effect we should expect to find this difference yet more marked, since, as was

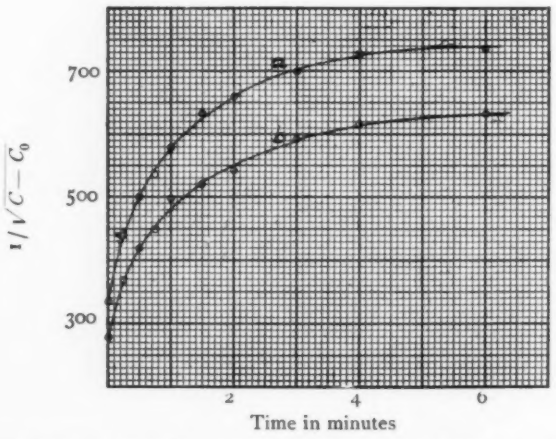
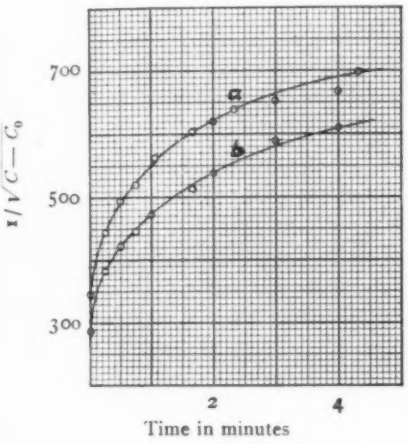


Fig. 17. Variation in rate of recovery with intensity of excitation. Cell C. Five seconds excitation by red light. Initial dark resistances: *a*, 9,760; *b*, 9,705.

shown in the case of white light, the effect of increasing intensity is to increase the rate of saturation for the first few minutes. This result corresponds to the fact earlier observed that for the cell in question the recovery for green light is more rapid than for red. (See Fig. 7.) A comparison of the time of saturation for red and infra-



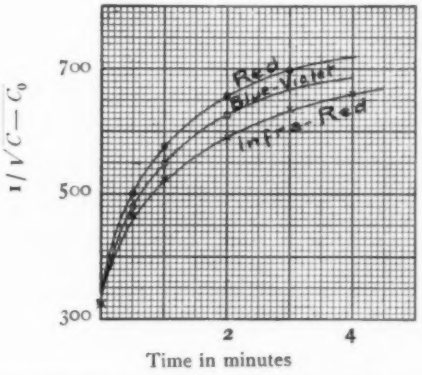


Fig. 18. Variation in rate of recovery with intensity of excitation. Cell C. Five seconds excitation by infra-red light. Initial dark resistances: *a*, 10,125; *b*, 8,040.

Fig. 19. Variation in rate of recovery with change in wave-length of light. Cell C.

red light of varying intensity, for the same cell, with the same apparatus, gave the results indicated in Table III.

For each color saturation was reached more quickly for the less intense light. A study of the percentages shows, however, that the rate of excitation for the more intense light is greater at first. This result is shown graphically for red light in the curves of Fig. 24, where the values of these ratios are plotted against the times. For exact results the denominator of each ratio should have been the change $(C - C_0)$ when complete saturation was attained but, as can be seen from the table, the error introduced by taking the 2½ hour ratio is slight and does not seriously modify the results. For the least intense light complete saturation was attained in one hour, hence the one hour ratio was used.

A comparison of the results for excitation by red and infra-red light at a distance of 50 centimeters, where the final effects produced

TABLE III.

Values of the Ratios, $(C_1 - C_0)/(C_2 - C_0)$, *for Different Times during Excitation.*
(*Cell I.*)

	Red.			Infra-red.	
	(100 cm.)	(50 cm.)	(25 cm.)	(50 cm.)	(25 cm.)
10 sec. / 2½ hrs.		.340	.449	.192[1]	.282
30 sec. / 2½ hrs.		.559	.601	.371	.457
1 min. / 2½ hrs.		.687	.655	.493	.572
5 min. / 2½ hrs.		.847	.766	.746	.741
10 min. / 2½ hrs.		.889	.813	.828	.807
30 min. / 2½ hrs.		.945	.894	.949	.906
1 hr. / 2½ hrs.		.968	.946	.992	.942
2 hrs. / 2½ hrs.		.992	.985	1.000	.986
10 sec. / 1 hr.	.197	.351	.475	.194	.299
30 sec. / 1 hr.	.372	.577	.636	.374	.486
1 min. / 1 hr.	.533	.710	.693	.497	.606
5 min. / 1 hr.	.860	.875	.809	.751	.787
10 min. / 1 hr.	.929	.918	.859	.834	.858
30 min. / 1 hr.	.987	.975	.945	.957	.962

The ratios of the final resistance to the initial resistance were for red light: 100 cm., 72.8 per cent.; 50 cm., 58.2 per cent.; 25 cm., 35.2 per cent. For infra-red: 50 cm., 59.8 per cent.; 25 cm., 41 per cent.

were approximately the same, shows that infra-red excitation proceeds more slowly than red. This again confirms the fact that recovery for infra-red light was found to be slower than for red. (See Figs. 7, 11, 15, 19.)

A rather curious result was obtained by dividing the ratios by the initial conductivity, which varied somewhat from time to time.

[1] The denominator in this case was the value of $C - C_0$ for two hours, the time at which the cell appeared to be saturated.

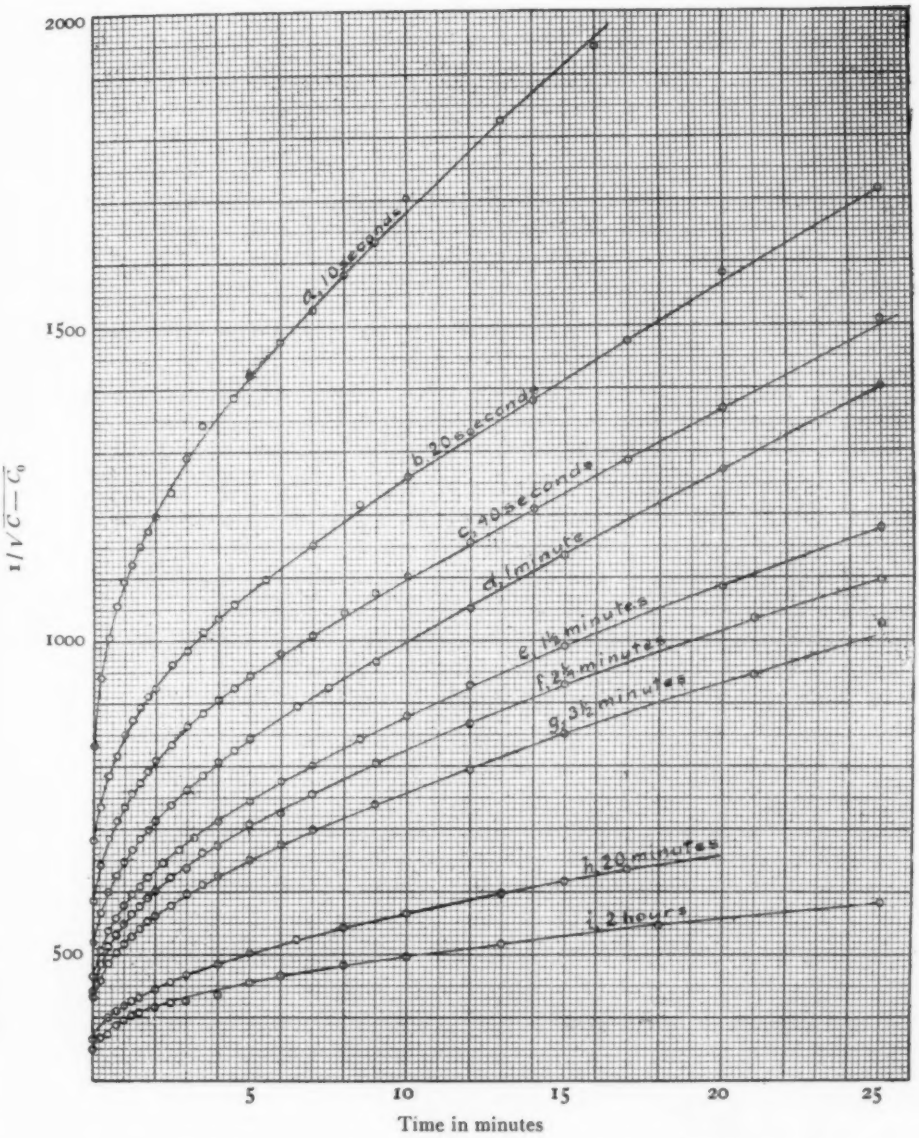


Fig. 20. Variation in rate of recovery with time of excitation. Cell I. Red light excitation. Initial dark resistances: *a*, 49,760; *b*, 55,160; *c*, 52,810; *d*, 52,870; *e*, 52,350; *f*, 53,120; *g*, 52,380; *h*, 51,480; *i*, 52,200.

These ratios, given in Table IV., will be seen to be almost identical for infra-red excitation which reduced the resistance to 41 per cent. and red which reduced it to 58 per cent. of the dark resistance.

TABLE IV.

Values of the Ratio, $\frac{C_1 - C_0}{\frac{C_2 - C_0}{C_0}}$, *for Different Times during Excitation by Red and Infra-red Light.* (*Cell I.*)

	Red.			Infra-red.	
	(100 cm.)	(50 cm.)	(25 cm.)	(50 cm.)	(25 cm.)
10 sec. / 2½ hrs.	10,500	17,050	26,600	9,250	16,950
30 sec. / 2½ hrs.	19,600	28,000	35,500	17,800	27,500
1 min. / 2½ hrs.	28,400	34,400	38,800	23,700	34,400
5 min. / 2½ hrs.	45,800	42,500	45,300	35,900	44,600
10 min. / 2½ hrs.	49,500	44,600	48,100	39,900	48,600
30 min. / 2½ hrs.	52,500	47,400	52,900	45,700	54,500

The initial values of the conductivity were : red, 100 cm., 1/53,250; 50 cm., 1/50,100; 25 cm., 1/59,150; infra-red, 50 cm., 1/48,100; 25 cm., 1/60,150.

The experiments with Cell A were with the same apparatus except that for the green excitation the metal screen was removed and the light of the acetylene flame allowed to fall directly on the green glass window in order to obtain an effect comparable with that for the red. The results are shown in Table V. and Fig. 25.

These results differ from the results for Cell I. much as the recovery curves for the two cells differ. Apparently the time in which saturation is attained is very nearly independent of the intensity of excitation, but, as in the case of Cell I., the rate of excitation for the first minute increases rapidly with the intensity. A comparison of the results for green, and for red excitation at 100 centimeters, which gave approximately the same reduction in resistance indicates that the excitation by red light is more rapid than that by green light except for the first few seconds. These results agree fairly closely with the results obtained for recovery (see Fig. 11).

TABLE V.

Values of the Ratio, $(C_1 - C_0)/(C_2 - C_0)$, *for Different Times during Excitation.*
(*Cell A.*)

	Green.	Red.			Infra-red.		
	(25 cm.)	(100 cm.)	(50 cm.)	(25 cm.)	(100 cm.)	(50 cm.)	(25 cm.)
10 sec. / 2½ hrs.	.325	.208	.453	.569	.165	.318[1]	.423
30 sec. / 2½ hrs.	.427	.371	.620	.691	.294	.449	.553
1 min. / 2½ hrs.	.494	.490	.722	.752	.382	.558	.629
5 min. / 2½ hrs.	.643	.726	.856	.863	.632	.737	.779
10 min. / 2½ hrs.	.705	.795	.886	.896	.729	.794	.835
30 min. / 2½ hrs.	.793	.881	.928	.934	.840	.898	.910
1 hr. / 2½ hrs.	.853	.929	.963	.962	.914	.949	.951
2 hrs. / 2½ hrs.	.951	.985	.988	.988	.983	1.000	.992
10 sec. / 3½ hrs.	.303	.203	.443		.159		
30 sec. / 3½ hrs.	.397	.359	.612		.283		
1 min. / 3½ hrs.	.460	.478	.711		.368		
5 min. / 3½ hrs.	.598	.709	.838		.608		
10 min. / 3½ hrs.	.657	.777	.866		.701		
30 min. / 3½ hrs.	.740	.859	.907		.808		
1 hr. / 3½ hrs.	.794	.907	.941		.879		
2 hrs. / 3½ hrs.	.885	.963	.967		.945		
2½ hrs. / 3½ hrs.	.931	.975	.978		.962		
3 hrs. / 3½ hrs.	.965	.991	.992		.980		

The ratios of the resistance attained in 2½ hours to the initial resistance were: green, 45.8 per cent.; red, 100 cm., 46.9 per cent.; 50 cm., 26.4 per cent.; 25 cm., 17.5 per cent.; infra-red, 100 cm., 51.2 per cent.; 50 cm., 27.7 per cent.; 25 cm., 23.5 per cent. The ratios for 3½ hours were: green, 43.2 per cent.; red, 100 cm., 46.3 per cent.; 50 cm., 25.9 per cent.; infra-red, 100 cm., 50.2 per cent.

[1] The denominator for the ratios of this column was $C - C_0$ for two hours.

While the form of the saturation curve was not determined for other cells it was noted in connection with the experiments upon sensibility and wave-length that the rate of saturation of Cell B for infra-red light was decidedly slower than for red, green, or violet light. Whereas in the exposure to visible light the greatest effect

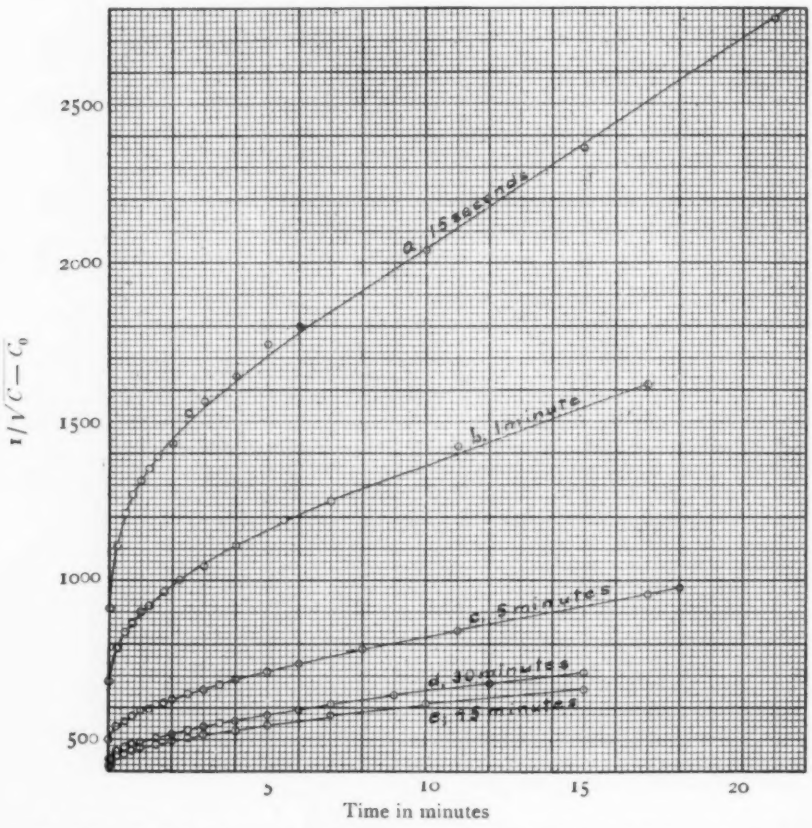


Fig. 21. Variation in rate of recovery with time of excitation. Cell I. Green light excitation. Initial dark resistances: *a*, 54,080; *b*, 52,450; *c*, 52,140; *d*, 51,710; *e*, 54,020.

was obtained during the first three seconds, when the cell was exposed to infra-red light the resistance continued to fall with fair regularity during the entire thirty seconds.

From the results of the experiments upon recovery and excita-

tion it is evident that there is a marked difference in the behavior of selenium towards light of different wave-lengths and that this behavior varies with the individual cell. May this difference not be due to the difference in the penetrating powers of the light of varying intensities and wave-lengths?

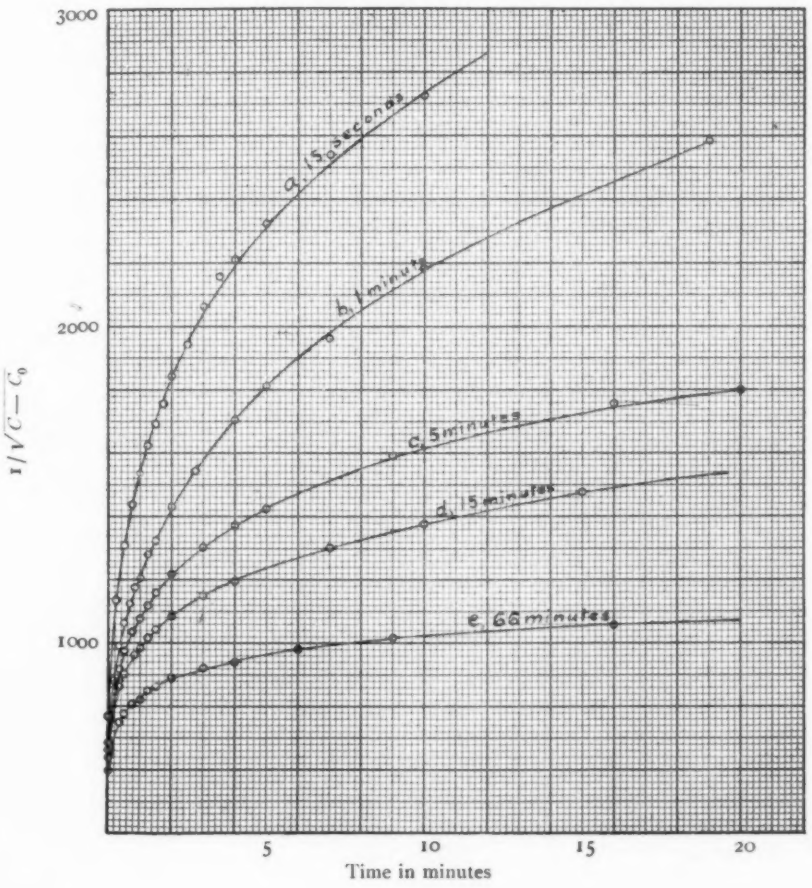


Fig. 22. Variation in rate of recovery with time of excitation. Cell II. Red light excitation. Initial dark resistances: *a*, 58,630; *b*, 58,500; *c*, 61,320; *d*, 60,160; *e*, 62,840.

Resistance and E.M.F.

In order to compare the relative behavior of the cells studied in other respects than towards color the dark resistance of the cells

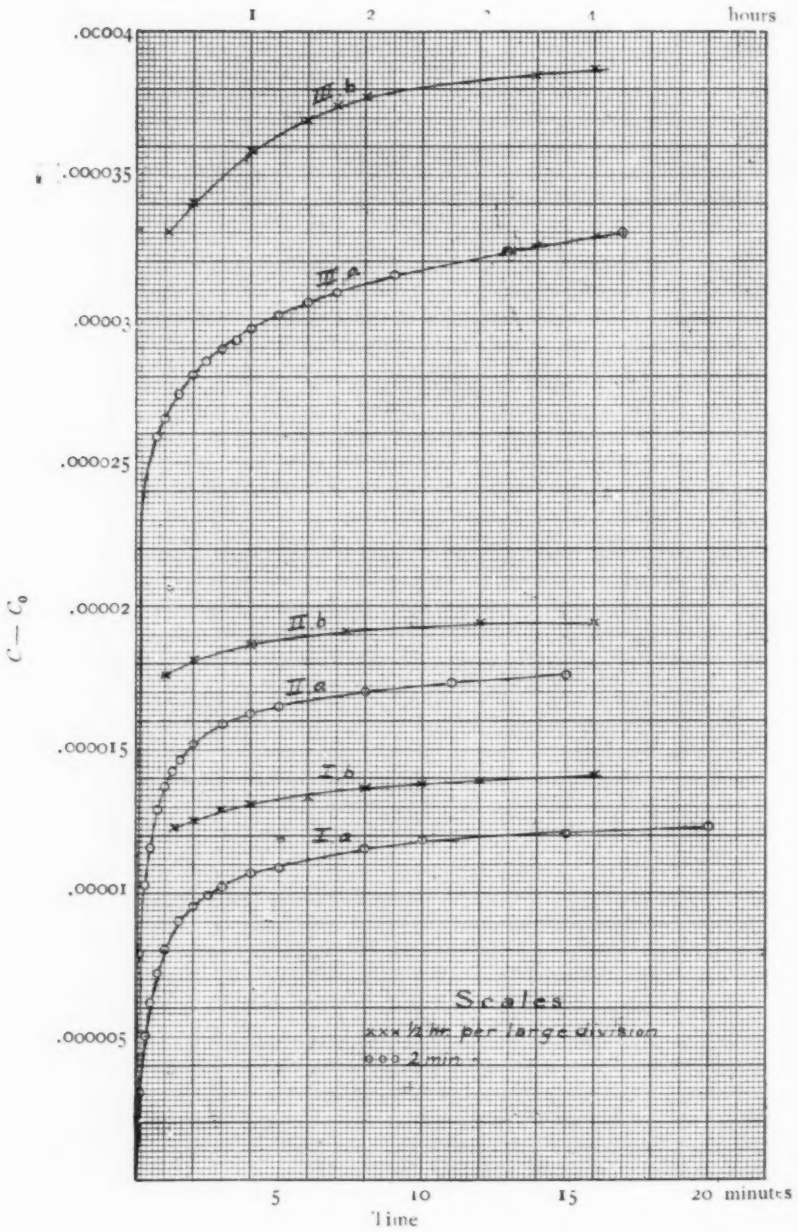


Fig. 23. Saturation curves for different intensities of excitation. Cell I. Distance of exciting light. I., 100 cm.; II., 50 cm.; III., 25 cm. Initial dark conductivities: I., .0000190; II., .0000205; III., .0000198.

was measured when currents of different strengths were sent through them. Currents ranging from that due to a single storage cell to that due to five cells in series were passed through each cell and the corresponding resistances noted. To compensate for any errors due to increase of temperature or permanent changes produced in the cells by the stronger currents, measurements were taken for

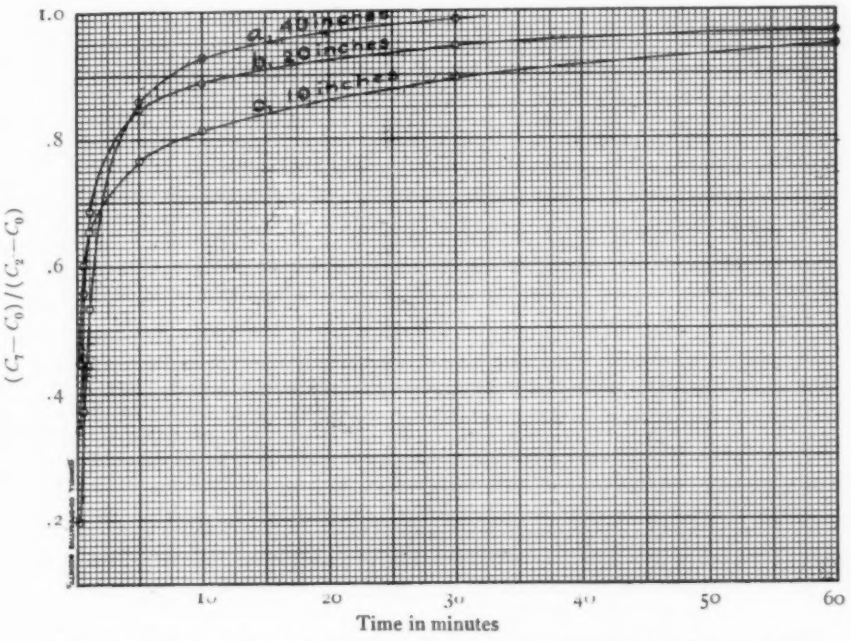


Fig. 24. Relation of rate of excitation to intensity of exciting light. Cell I. Red light excitation.

both increasing and decreasing E.M.F.'s. Curves plotted with measured resistances as ordinates and E.M.F.'s as abscissæ (see Fig. 26) are almost straight lines of similar slope. The two return curves which are of decidedly different slope may probably be explained by a change in temperature as both were for sensitive cells. These results lead to no conclusion regarding variation in cell behavior.

Sensibility and Wave-length.

As a test for the variation in the sensibility of the cells for the different wave-lengths, the spectrum formed by a Nernst glower

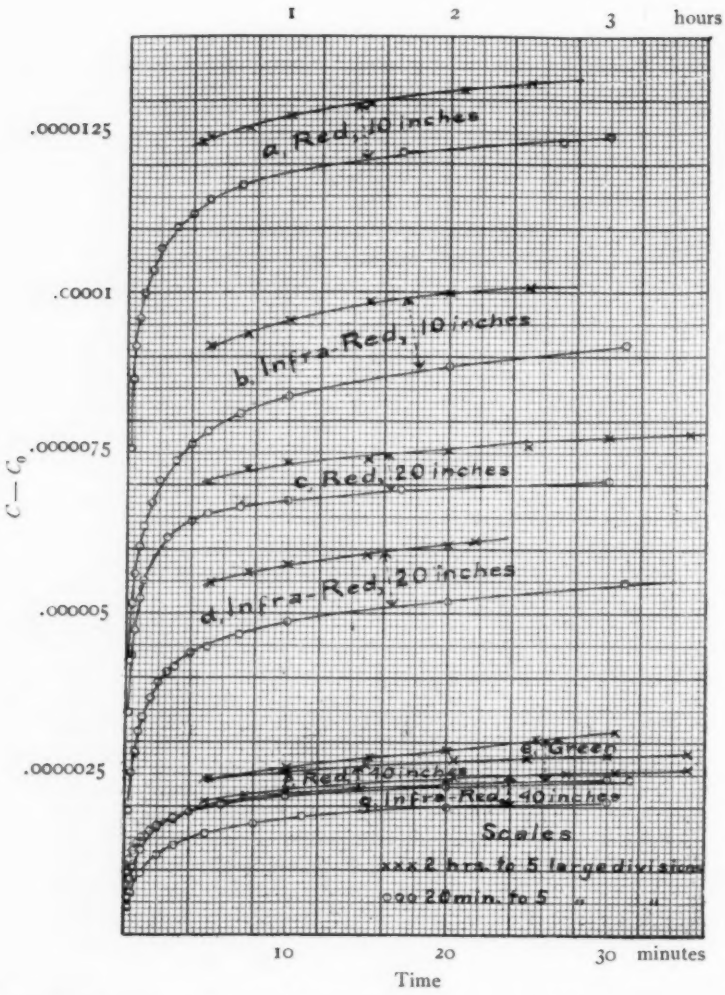


Fig. 25. Saturation curves. Cell A. Initial dark conductivities: *a*, .00000281; *b*, .00000310; *c*, .00000274; *d*, .00000233; *e*, .00000259; *f*, .00000244; *g*, .00000261.

was used. With five-minute intervals for rest, the cell was excited successively by violet, green, red and infra-red light and the resistance before and after excitation measured by the usual Wheatstone bridge method. Then after a ten-minute interval the process was repeated in the reverse direction from infra-red to violet. This

method is of course open to the objection that the cell does not have time to recover between readings and therefore has its sensibility altered for the later readings. Furthermore since the intensity of excitation is different for the different wave-lengths, the stage of recovery depends upon the order of the colors used. Partially, at least, to obviate these difficulties a second set of observations was taken, after a rest of a day or more, in the reverse direction proceeding from infra-red to violet and back again to infra-red. Both sets of observations are plotted in Fig. 27. Since the spec-

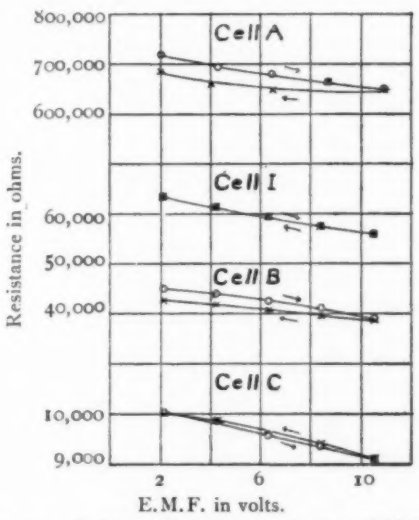


Fig. 26. Relation of resistance of cell to E.M.F.

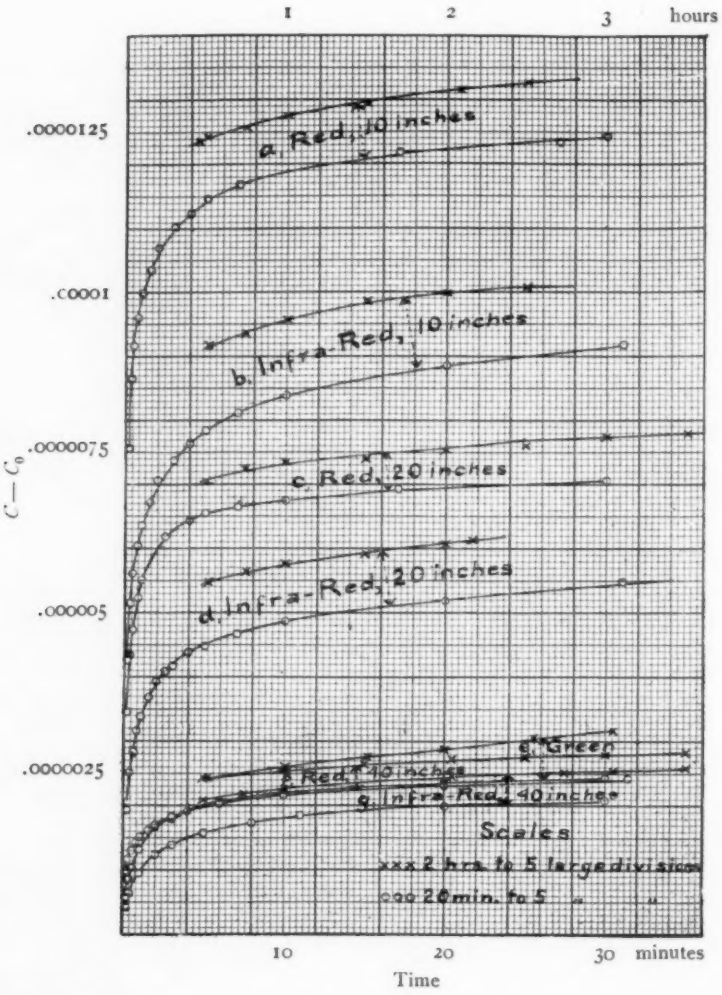
trum of the Nernst glower is much stronger in red rays, only the relative sensibility of the different cells is shown. Below are given briefly the results for each cell.

Cell A is far more sensitive to red light and infra-red than to violet and green and relatively more sensitive to infra-red than to red.

Cell I. gives similar results.

Cell B, on the contrary, is most sensitive for red and shows towards infra-red a sensibility little greater than towards green, while the sensibility towards green is much greater than in the two cells A and I.

Cell C displays the same order of sensibility as cells A and I.,



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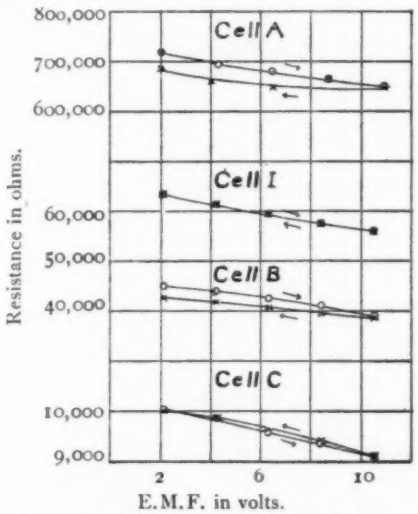

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Cell I. gives similar results.

Cell B, on the contrary, is most sensitive for red and shows towards infra-red a sensibility little greater than towards green, while the sensibility towards green is much greater than in the two cells A and I.

Cell C displays the same order of sensibility as cells A and I.,

but is in general less sensitive towards all colors. Moreover, there is not the wide difference between the sensibility towards red and green found in those cells.

Although these results show a difference in the behavior of the cells, the difference is not such as to explain the difference in the

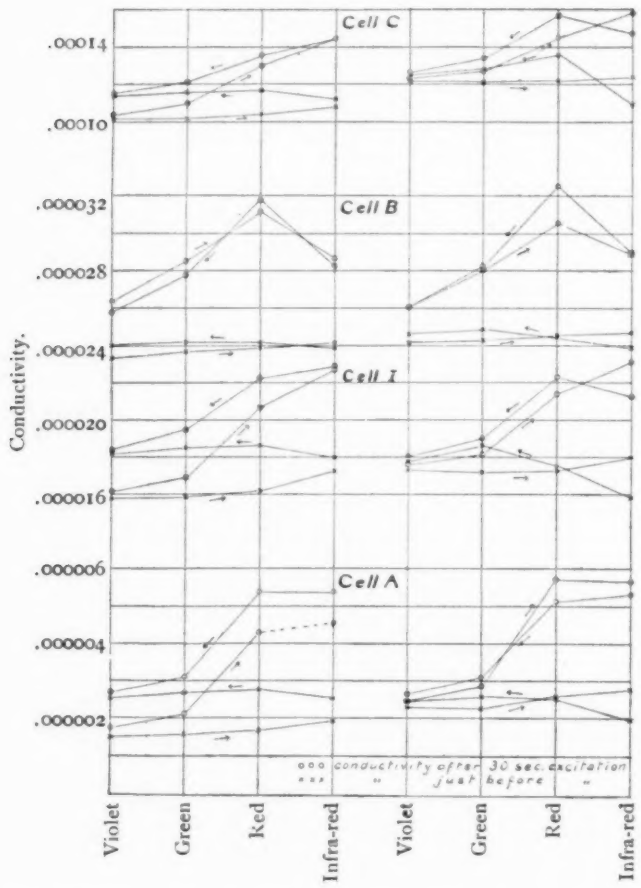

Fig. 27. Relative sensibility for light of different wave-lengths. Excitation by spectrum of Nernst lamp.

color curves obtained with the different cells. Incidentally it may be noted that the results contradict the statement of Marc[1] to the

[1] R. Marc, Zs. für anorg. Chemie, Vol. 37, p. 459.

effect that whereas continued blue lighting has a strong influence upon the sensitiveness towards red light, exposure to red light is without influence on the behavior towards blue light. Here previous exposure either to blue or red light is seen to affect the sensibility towards the other.

The Effect of Electric Oscillations.

In the study of Cell I. it was found that the recovery curves were markedly affected by the time of rest. A cell after three days in the dark showed a much more rapid recovery than after two days in the dark and that again a more rapid recovery than after one day in the dark, so that complete recovery was not effected in less than three days, perhaps not then. As a possible means of restoring the cell more quickly to standard condition, the effect of electric oscillations was tried. Previous experimenters who had studied the behavior of selenium towards electric oscillations had reached contradictory conclusions. Berndt[1] says that powerful electric waves lessen the resistance. Perreau[2] who tried the effect of electric oscil lations on a selenium cell obtained only negative results. Massini[3] concludes that electric waves do not have a sensible influence on the resistance of selenium. Any effect observable may be attributed to imperfect contacts and a coherer effect.

Since the conclusions were so uncertain it seemed worth while to

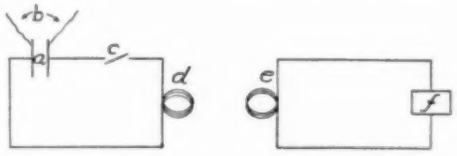

Fig. 28. *a*, Leyden jar; *b*, wires to induction coil; *c*, zinc spark gap; *d*, *e*, coils of three turns of large copper wire; *f*, selenium cell.

test the effect upon different cells. For this purpose a circuit was set up as in Fig. 28. The primary circuit consisted of a medium-sized Leyden jar, a quarter-inch zinc spark gap, and an inductance of three turns of heavy copper wire; the secondary, of a similar

[1] G. Berndt, Beiblätter, Vol. 29, p. 288.
[2] M. Perreau, Comptes Rendus, Vol. 129, p. 956.
[3] A. Massini, Nuova Cimento (5), Vol. 1, p. 358.

inductance connected directly to the cell. The distance between primary and secondary coils was varied from three to twelve inches. Cell I. was first excited strongly for one minute by red light, then placed in the dark and the resistance noted after a definite interval, say 5 minutes. Again the cell was excited, disconnected from the bridge and subjected to oscillations for two minutes. After the same interval the resistance was measured and found to be greater than in the first case. The recovery had apparently been hastened by the oscillations, but the resistance was not at once restored to the initial value. Further trials with varying distances between primary and secondary gave the following results. Very strong oscillations applied after the recovery had been in progress for some minutes caused the resistance at first to increase almost to the initial value and then to decrease by sudden leaps until in a few minutes it had attained very nearly the value given it by exposure to red light. In this condition the cell was comparatively insensitive to light. Subjection to weaker oscillations again hastened recovery as in the first instance. When the cell after subjection to this treatment was left in the dark twenty-four hours it showed a decrease in resistance greater than that due to red light and was again relatively insensitive to light. Subjection to oscillations again increased the resistance. By repeated trials a strength of oscillations was found such that the resistance was restored to its initial value and kept comparatively constant. Exposure for fifteen seconds was sufficient to produce this result. When this strength had been found, a set of color curves was taken in which the cell was subjected to oscillations between each two exposures to light. This set was found to agree fairly closely with curves taken in the usual manner with a twenty-four hour rest between exposures.

The high resistance cell, A, of great sensibility, was next tried and the oscillations found to produce upon it no immediately apparent effect. Cell B, of the Ruhmer type, which had earlier given some trouble because of a slight inconstancy in the dark resistance such as might be due to poor contacts, had its dark resistance markedly increased and rendered constant by subjection to oscillations. After treatment the cell was also somewhat more sensitive than before. After this initial effect the oscillations exerted no fur-

ther observable influence, but the dark resistance maintained its higher value. Tests of three other cells showed either no effect or only a very slight hastening of the earlier stages of decay.

After these experiments were concluded the cells chanced not to be used for nearly two months. When they were again tested some peculiar results were noted. The dark resistance of Cell I. had fallen from approximately 50,000 ohms to 1,290 ohms. This resistance was unchanged at the end of twenty-four hours, when electric oscillations applied for fifteen seconds raised the resistance approximately to 60,000 ohms. During the succeeding two weeks, when the cell was excited for several hours nearly every day, the dark resistance varied between 48,000 and 60,000 ohms, apparently unaffected by a renewed application of oscillations. Cell A had previously been apparently uninfluenced by the oscillations, but its dark resistance after two months was found to have fallen to 50,000 ohms, and to be extremely unsteady. Fifteen seconds treatment with oscillations brought the resistance at once up to 430,000 ohms, where it remained for several minutes. The cell then responded to light as usual and recovered its normal dark resistance in the ordinary manner. When again tested with oscillations it was apparently uninfluenced. Cell B also showed a diminution of resistance from 50,000 ohms to 25 ohms. As the current from the single storage cell was a second time sent through the cell, the resistance for no apparent reason jumped to 3,130 ohms, but at this value it remained steady despite rapid reversals of the current through the cell. A 16-c.p. incandescent lamp at a distance of four inches reduced this resistance to 2,900 ohms, from which it slowly rose to 4,600 ohms in eighteen hours. Oscillations applied for ten seconds increased the resistance to 51,650 ohms. After six hours its resistance had decreased 2,000 ohms. Cell C when tested after two months of rest showed no influence of the oscillations. Its resistance was 8,650 ohms, a normal dark resistance for the cell.

These results appear rather contradictory. Evidently we have to deal with two effects, the immediate and the deferred. Both actions depend probably upon the condition of the cell. If the cell is in its most sensitive condition it will probably appear unaffected, but if allowed to rest in the dark for any appreciable length of time after

treatment will gradually lose resistance and sensibility. A cell which is not in its most sensitive condition may have both resistance and sensibility increased by the action, but this increase may be only temporary. A possible explanation of the immediate results might be that the action is in the nature of a coherer effect; that the extreme slowness of the recovery of Cell I. is due to the massing together of particles which cling together long after the excitation has ceased. The oscillations by causing rapid vibration of the particles break up these aggregations and thus restore the cell to its normal condition. This hypothesis would serve to explain the increased value of resistance of Cell B, which may have had its particles so massed together that the aggregations would only partially break up and immediately reform, thus causing the vibrations in the dark resistance. Under the action of the electric oscillations these aggregations were broken up and the cell brought to its normal dark resistance. This explanation is borne out by the fact that when Cell B was excited by light the vibrations ceased and it was only as the dark resistance was approached that the unsteadiness was manifested. Evidently if this explanation is correct a cell, such as A, upon which no immediate effect was shown would be one in which the particles show little tendency to cohere after the excitation has ceased. A possible objection to this explanation is that sudden jars were not found to produce a similar effect. Other investigators, however, have found such an effect. Hesehus notes that a sudden blow hastens the recovery of the cell after excitation and has exactly the same effect as upon the residual charge in a Leyden jar.[1] This explanation agrees also with the statement sometimes made, to the effect that an alternating current applied to a new cell will increase its sensitiveness. Fritts, for instance, says that the resistance of his cells is raised by sending an alternating current through them and that they immediately become extremely sensitive to light, currents and other influences. Occasionally they drop back.[2]

Just how to account for the deferred effect upon this hypothesis is not so evident. Since it is the more powerful oscillations which produce this effect the more quickly, and since it may be produced

[1] N. A. Hesehus, Phys. Zs., Vol. 7, p. 163.
[2] C. E. Fritts, Elec. Rev., Vol. 16, p. 208.

in a cell in a highly sensitive condition, possibly the increase of motion due to the oscillations may cause particles already in motion to strike each other with such force as to cohere, and the aggregations once started, although at first too small to have a noticeable effect upon the conductivity, continue to attract other particles until we have a marked coherer effect.

Summary.

The results of the investigation may be summarized as follows:

I. The form of the recovery curve depends on the intensity of the excitation and on the wave-length of the exciting light. In general the time of recovery increases with the intensity of the excitation. Recovery is slower for excitation by infra-red light than by light of the visible spectrum. Individual cells differ in the relative time of recovery for different wave-lengths as well as in the absolute time of recovery for light of the same intensity and wave-length.

II. The effect of increased duration of excitation upon the form of the recovery curve is similar to the effect of increased intensity.

III. Excitation requires time. While from 60 per cent. to 80 per cent. of the entire increase in conductivity may be produced in the first five minutes, for complete saturation hours are required. The rate of excitation varies with the intensity and the wave-length of the exciting light. The time required for complete saturation may decrease as the intensity of the light diminishes, but the initial rate of excitation always increases with the intensity. The variation in the rate of excitation of the individual cells for the different wave-lengths corresponds to the variation in their rate of recovery, *e. g.*, those cells which show the most rapid recovery for red light show also the most rapid excitation for red light.

IV. The relation between the resistance and E.M.F. is approximately a straight line relation. The resistance decreases as the E.M.F. increases.

V. Different cells show a variation in the relative sensibility to light of different wave-lengths, but no relation was found between this variation and the variation in the curves of recovery and excitation.

VI. Electric oscillations affect the resistance and hence the sensi-

bility of the selenium cell. These effects are of two kinds, immediate and deferred. Both may be in the nature of coherer effects and both depend upon the condition of the cell at the time when the oscillations are applied.

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ON THE ROTATION OF CONSTANT DEVIATION PRISMS.

By H. S. Uhler.

IN each of the years 1905 and 1908 the writer has had occasion to set up a specimen of Hilger's constant deviation spectroscope and, in so doing, has experienced much satisfaction because of the excellence of the construction of these pieces of apparatus. A careful study of the various adjustments of this type of spectroscope led to the conclusion that the position of the axis of rotation of the prism table with respect both to the prism and to the axes of the collimator and telescope was a matter of greater importance than might be supposed at first glance. If, for example, the axis of rotation is not correctly placed the emergent beam from the prism will not illuminate the telescope objective symmetrically and hence the waves of light inside the telescope will not converge symmetrically to the focus at the middle of the field of view. When this is the case, the medial lines of the extra-focal images on the two sides of the true focal plane will lie on opposite sides of the optic axis of the telescope, so that, if the observer is not very careful to focus correctly, the settings of the cross-hairs on the centers of these images will cause the readings of the wave-length drum to be a little too large or slightly too small. This inconvenience would practically vanish if the lenses ordinarily furnished with spectroscopes of moderate price were sensibly achromatic for the entire visible spectrum from about 400 $\mu\mu$ to about 800 $\mu\mu$. For then the instrument would remain in perfect adjustment for all visible radiations if it were correctly focused for some one wave-length. However, since such ideal conditions are not realized, in general, it is desirable to place the axis of rotation in the best possible position so as to cause the telescope objective to be illuminated as symmetrically as is consistent with a pure rotation of the prism.

In addition to the preceding remarks, the following arguments

may be adduced to show the desirability of having at one's disposal a set of comparatively simple formulæ which give the coördinates of the axis of rotation of the prism table in terms of known physical constants. It would be easier and far more accurate for the manufacturer, who has to deal with different sorts of glass, to substitute his numerical data in the formulæ than to determine anew and experimentally the position of the axis of rotation for each kind of glass and for each prism angle. Again, it is not very difficult to modify an ordinary spectrometer so as to convert the apparatus into a constant deviation spectroscope by replacing the triangular prism by a suitable quadrilateral prism and, if necessary, by displacing the collimator parallel to itself. The resulting spectroscope

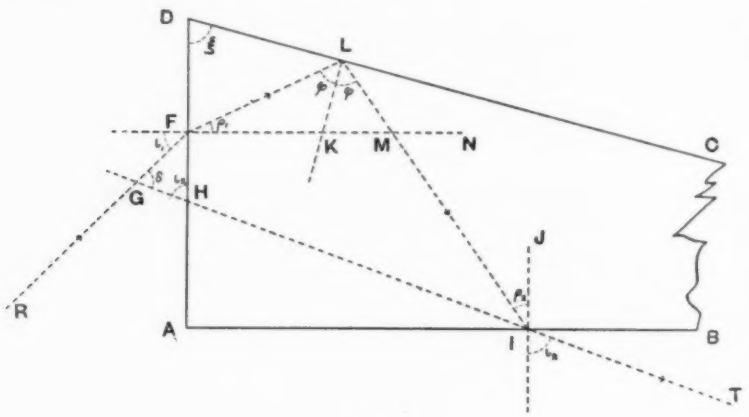


Fig. 1.

would not be comparable with Hilger's instruments because of the absence of the drum, but, when the circular scale was once calibrated by the aid of lines of known wave-length, the altered spectrometer would have several important advantages over the unmodified form. In order to properly remodel a piece of apparatus, in the manner just indicated, it would be unquestionably convenient to have the necessary elementary theory worked out in advance in an easily accessible article.

Since the writer has sought in vain to find a complete discussion of the quadrilateral prism in any practical journal or in any modern text-book on spectroscopy, and since he has been informed by

Hilger both that the manufacturers are not aware of the existence in print of such a discussion and that it would be desirable to fill up this gap in the literature of prisms, the writer has decided to present formally in the following pages the results of his investigation of the properties of the quadrilateral prism.

In Fig. 1 $ABCD$ outlines a portion of a principal section of the type of prism under consideration. The earliest account of a special case of this prism is given by Pellin and Broca.[1] The angle BAD equals 90° and the angle ADC may have, in general, any value between 45° and 90°. Since we shall only be concerned with the deviation of trains of plane waves and with isotropic media the use of rays is permissible. A general expression for the total deviation of any incident ray, $\overline{RF}$, will now be derived. The path of the light is indicated by the broken line $RFLIT$. The normals to the faces $\overline{AD}$, $\overline{DC}$ and $\overline{BA}$ at the points of deviation F, L and I are denoted respectively by $\overline{FN}$, $\overline{LK}$ and $\overline{IJ}$. The angles to which the various symbols refer may be readily seen by inspecting Fig. 1.

$\angle FHG = \angle JIH = \epsilon_2$, hence, from the triangle FGH, it is evident that the total deviation δ is given by

$$\delta = 90° + \epsilon_1 - \epsilon_2. \tag{1}$$

$\angle MKL = \xi$, since the sides of these angles are mutually perpendicular, etc., and also $\angle MKL = \rho_1 + \varphi$, from the triangle FKL, therefore

$$\rho_1 = \xi - \varphi. \tag{2}$$

Again, $90° + \rho_2 = \angle BIM = \angle NML = \xi + \varphi$, from the triangle KML, hence

$$\rho_2 = \xi + \varphi - 90°. \tag{3}$$

Combining equations (1), (2) and (3) with the relations $\sin \epsilon_1 = n \sin \rho_1$ and $\sin \epsilon_2 = n \sin \rho_2$, where n denotes the refractive index of the material of the prism relative to the surrounding medium, it results that

$$\delta = 90° + \sin^{-1} [n \sin (\xi - \varphi)] - \sin^{-1} [n \sin (\xi + \varphi - 90°)]. \tag{4}$$

When $\xi - \varphi = \xi + \varphi - 90°$ or $\varphi = 45°$, $\delta = 90°$ independently of n and ξ. But then $\rho_1 = \xi - 45°$ and $\rho_2 = \xi - 45°$ so that

[1] Ph. Pellin and André Broca, Jour. de Phys. (3), Vol. 8, p. 314, 1899.

$\rho_1 = \rho_2$. The equality of these angles in the case of a simple triangular prism is, as is well known, the necessary and sufficient condition for minimum deviation and, since the process of total internal reflection at the face $\overline{CD}$ does not alter the focal properties of the waves, it follows that when the wave-normal is deviated by 90° at internal reflection the emergent waves possess all the important properties which they would have acquired on traversing a triangular prism at minimum deviation. But, as we have just seen, when $\varphi = 45°$ the total deviation has the constant value 90° irrespective of n, that is, of the wave-length of the radiations concerned. These last facts make the Pellin and Broca prism more convenient for use in simple spectroscopes than an ordinary triangular prism of equal resolving power since, in the case of the former, it is only necessary to rotate the dispersing system, while the telescope remains fixed at the constant angle 90° with respect to the collimator, in order to bring waves of each wave-length successively to a focus at the cross-hairs under conditions most favorable to good definition.

If 2τ denotes the refracting angle of a triangular prism then, when minimum deviation prevails, $\rho_1 = \rho_2 = \tau$ so that a Pellin and Broca prism of angle ξ is equivalent to a triangular prism of angle 2τ when $\xi - 45° = \tau$. For this reason it is customary in elementary textbooks to analyze a 75° quadrilateral prism into the halves of a 60° triangular prism and a total reflecting prism of angles 45°, 45° and 90°.[1] Obviously, the rays are not literally subjected to a minimum of deviation by the quadrilateral prism and furthermore this prism differs from the equivalent triangular prism in the relatively unimportant matter of absorption. Again, in order to prevent the fourth face of the Pellin and Broca prism from interfering, by reflection or otherwise, with the passage of light whose wave-length differs a little from that of the radiations which traverse the prism at effective minimum deviation, it is desirable in practice to make the faces $\overline{DC}$ and $\overline{AB}$ somewhat longer than would result from the geometrical synthesis of the prism from the three hypothetical triangular prisms mentioned above. For example, in one of Hilger's spectroscopes the face $\overline{AD}$ equals 37.3 mm. and the faces $\overline{DC}$ and $\overline{AB}$ exceed the

[1] See E. C. C. Baly's Spectroscopy, p. 58.

minimum lengths 44.6 mm. and 60.9 mm. by about 15 mm. and 5.7 mm. respectively.

In what precedes, it has been assumed that total internal reflection takes place at the plane $\overline{CD}$ and that a finite fraction of the light emerges from the face $\overline{AB}$. These hypotheses cause restrictions to be placed upon the values of n and τ. At effective minimum deviation $\varphi = 45°$ and hence the index of refraction must be greater than $\sqrt{2}$ (= 1.4142), in order that total reflection may occur. Again, for emergence from $\overline{AB}$, when $\varphi = 45°$, the inequality $\tau < 45°$ must be satisfied and from this fact, or from the circumstances which must prevail at incidence at the face $\overline{AD}$, it follows that the index of refraction must not exceed $(\sin \tau)^{-1}$.

Table I. may serve to give in concrete form an idea of the dependence of n and ϵ_1 upon τ when all the preceding limitations are taken into account and $\varphi = 45°$ or $\rho_1 = \rho_2 = \tau$.

TABLE I.

2τ	τ	Inferior Limit of n, $= \sqrt{2}$.	Superior Limit of n, $= (\sin \tau)^{-1}$.	Inferior Limit of ϵ_1, $= \sqrt{2} \cdot \sin \tau$.
40°	20°	1.4142	2.9238	28° 55′ 36″
50	25	"	2.3662	36 42 12
60	30	"	2.0000	45 0 0
70	35	"	1.7434	54 12 34
80	40	"	1.5557	65 22 22
85	42.5	"	1.4802	72 49 45
89.8	44.9	"	1.4167	86 36 47

As a practical example, Jena glass number *O.* 165 has an index of refraction 1.7464 for *C* and hence a quadrilateral prism having $\tau = 35°$ and made of this kind of glass could not be used to best advantage in the visible spectrum from a little "below" *C* to the ultra-violet limit. For *A′*, $n = 1.7380$.

Again, Jena glass number *S.* 57 is not suitable for use in a quadrilateral prism having $\tau = 30°$ which angle is, for well-known reasons, most frequently met with in practice. The indices of refraction for *A′*, *C*, *D*, *F* and *G′* respectively being 1.9349, 1.9493, 1.9626, 1.9981 and 2.0306, while the superior limit is 2.0000.

The inferior limit of n, $\sqrt{2}$, is of little practical importance since the least index of refraction of the media of which prisms designed

for visual or photographic purposes are ordinarily made exceeds this value.

As explained above, a suitable rotation of the quadrilateral prism between the objectives of a collimator and telescope, whose optic axes intersect at right angles, will cause the transmitted radiations to come to a focus successively upon the intersection of the cross-hairs under the very desirable conditions pertaining to minimum deviation. However, in order that the axis of the emergent beam from the prism may be made to coincide as nearly as possible with the optic axis of the telescope throughout the entire region of the spectrum, for the study of which interval the instrument is especially designed, it is essential to know the coördinates of the axis of rotation with respect both to the prism and to the optic axes of the collimator and telescope. The reasons why this coincidence should be approximately fulfilled have been suggested in an earlier paragraph.

Of course, the ideal method of procedure would be to actuate the prism by a mechanism which would constrain the center of the face $\overline{DA}$ and the effective center of the face $\overline{AB}$ to remain continuously in the optic axis of the collimator and of the telescope respectively. Theoretically this could be accomplished by any device which would be equivalent to attaching the prism to the movable beam of the type of mounting designed by Rowland for his concave grating spectrograph. The carriage tracks would then be parallel to the optic axes of the collimator and telescope. The practical realization of such a scheme which would produce absolutely perfect accuracy of adjustment would probably entail too great a complication of parts of the apparatus and hence would cause the commercial aspect of the matter to be impracticable. Furthermore, as will be shown later on, a sufficiently high degree of accuracy can be attained by a pure rotation of the prism around a fixed axis. In any particular case, the best position for this axis can be found either by actual trial or by elementary mathematical analysis. Since the substitution in comparatively simple formulæ of constants whose values have to be known in order to design the rest of the system advisedly is a perfectly general process, capable of any required degree of accuracy, the latter plan seems preferable to the former. Hence, we shall now take up the main object of this paper, which

is to derive and discuss all formulæ necessary for finding the position, with respect to known lines, of the axis of rotation of any quadrilateral prism of the type under consideration. Moreover, since the problem in hand is a purely geometrical one, no reference will be made to questions of optical and spectroscopic resolving power, of purity, of achromatism, etc. In other words, it will be assumed throughout that all parts of the apparatus are as perfect as possible and that they are in accurate adjustment.

In Fig. 2 the fixed coördinate axes $\overline{OX}$ and $\overline{OY}$ represent the optic axes of the telescope and collimator respectively. The light waves proceed in the direction $\overline{YO}$ before incidence at the first prism face and along $\overline{OX}$ after emergence, that is, they move in the negative direction along the axis of ordinates and in the positive direction

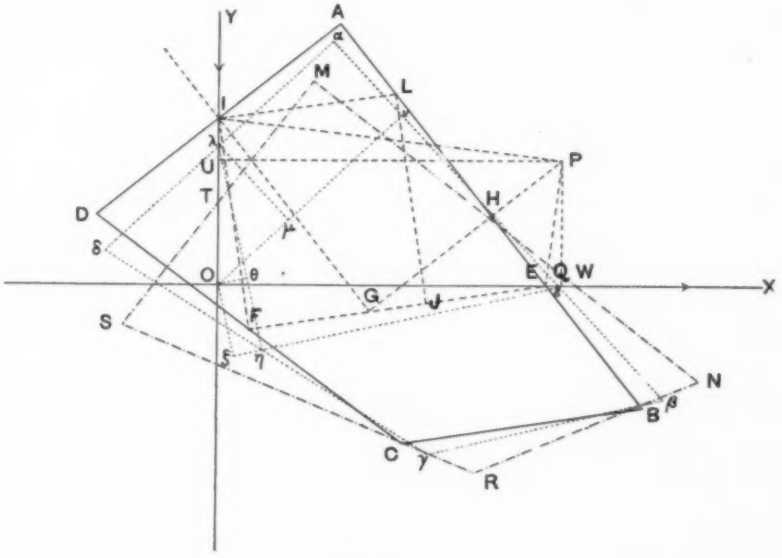


Fig. 2.

along the axis of abscissæ. Let P be the point of the axis of rotation which lies in the plane $\overline{XOY}$. The coördinates $\overline{OQ}$ and $\overline{OU}$ of P, with respect to the fixed axes, will be symbolized by x_0 and y_0. The lines of intersection with the plane $\overline{XOY}$ of the two faces of the prism which are mutually perpendicular will be used as a set of coördinate axes of variable position. The axis lying in the emergence

face will be considered as the axis of abscissæ and hence the projection of the incidence face will be the axis of ordinates. The positive direction of the variable axis of abscissæ will be measured from the variable origin along the actual surface of the material prism, whereas the positive direction of the variable axis of ordinates will be reckoned from this origin along the geometrical extension of the incidence face. In one position of the prism A is the variable origin, $\overline{AH} = x'$ and $\overline{HP} = y'$. For sake of brevity the phrase "the center of the emergence face" will be used to denote the point where the axis of a narrow pencil of light meets the emergence face when this axis intersects the incidence face at the geometrical center of the latter and when the light has passed through the prism at effective minimum deviation. Obviously, rotation of the prism about the axis through P can only cause the centers of the incidence and emergence faces to lie simultaneously in the optic axes of the collimator and telescope respectively for two positions of the prism, at most. In Fig. 2 these two orientations of the prism are indicated by $\overline{ABCD}$ and $\overline{MNRS}$. Since the problem in hand is to so determine the four coördinates of the axis of rotation as to cause the axis of the emergent beam to depart as little as possible from coincidence with the optic axis of the telescope, within a given angular interval, it is evident that this condition can be realized best by including the two positions $\overline{ABCD}$ and $\overline{MNRS}$ in the given interval. This angular interval depends upon factors with whose choice we are not here directly concerned, namely, upon the limits of the region of the spectrum for the study of which the instrument is designed and hence upon the indices of refraction of the material of the prism corresponding to the ends of this region.

Therefore, we shall derive formulæ first for x_0, y_0, x' and y' in terms of the width b of the incidence face, of the angle τ, and of the angles of incidence i_1 and i_2 corresponding respectively to the orientations $\overline{MNRS}$ and $\overline{ABCD}$. A later step in the argument will be to find a connection between the angles of incidence pertaining to the ends of the given interval of rotation and the angles i_1 and i_2.

The points T and I represent the first and second positions on the axis OY of the center of the incidence face and likewise the points W and E show the simultaneous locations on the axis $\overline{OX}$

of the center of the emergence face. In order to find the point P around which the line joining the centers of these faces must be rotated so as to move this line from the position $\overline{TW}$ to $\overline{IE}$ it is only necessary to construct the perpendicular bisectors of the segments $\overline{EW}$ and $\overline{TI}$. The middle points of $\overline{EW}$ and $\overline{TI}$ are denoted by Q and U in Fig. 2 and hence the perpendicular bisectors are the lines QP and UP. In the diagram $i_1 > i_2$ and $\overline{MS} = a\delta = \overline{AD} \equiv b$.

It is convenient to commence by finding an expression for the length of AE, that is, the distance from the edge through A to the center of the emergence face. The broken line $\overline{IFE}$ indicates the axis of a narrow pencil of light during the passage of the waves through the prism and therefore, since minimum deviation is alone under consideration, $\angle EFI = 90°$. Draw $\overline{IL}$ through I so as to make the angle AIL equal to τ. Also let $\overline{JL}$ be constructed perpendicular to $\overline{IL}$ and meeting $\overline{FE}$ at J.

$\overline{AE} = \overline{AL} + \overline{LE}$ and since $\angle JLE = \angle AIL = \tau$ we have $\overline{AE} = \frac{1}{2}b \tan \tau + \overline{JL} \sec \tau = \frac{1}{2}b \tan \tau + \overline{FI} \sec \tau$. Now, in the $\triangle DIF$, $\angle FDI = 45° + \tau$ and $\angle DFI = 45°$ hence $\overline{FI} : \sin (45° + \tau) = \frac{1}{2}b : \sin 45°$ or $\overline{FI} = \frac{1}{2}b (\sin \tau + \cos \tau)$, therefore

$$\overline{AE} = \tfrac{1}{2}b (1 + 2 \tan \tau). \tag{5}$$

We shall next derive four independent equations involving x_0, y_0, x' and y' from which these coördinates may be obtained separately. The angle of emergence equals the angle of incidence hence $\angle OEH = 90° - i_2$. But $\angle OEH + \angle HEP + \angle PEQ = 180°$ and $\angle PEQ = 90° - \frac{1}{2}(i_1 - i_2)$, consequently $\angle HEP = \frac{1}{2}(i_1 + i_2)$. Furthermore, $\angle IPE = \angle UPQ = 90°$ hence $\angle HPI = \angle HEP = \frac{1}{2}(i_1 + i_2)$. Now, in the $\triangle GPI$, $\overline{GP} = \overline{IG} \cot (\angle HPI)$ or $\overline{GH} + \overline{HP} = \overline{IA} + \overline{HP} = \overline{AH} \cot \frac{1}{2}(i_1 + i_2)$, whence

$$\tfrac{1}{2}b + y' = x' \cot \tfrac{1}{2}(i_1 + i_2). \tag{6}$$

Again, $x' = \overline{AH} = \overline{AE} - \overline{HE}$ so that by referring to equation (5) and to the $\triangle HEP$ we obtain

$$x' = \tfrac{1}{2}b(1 + 2 \tan \tau) - y' \cot \tfrac{1}{2}(i_1 + i_2). \tag{7}$$

In the $\triangle UPI$, $\angle UPI = \frac{1}{2}(i_1 - i_2)$ and $\overline{UP} = x_0$ therefore $x_0 = \overline{IP} \cos \frac{1}{2}(i_1 - i_2)$. Also, in the right $\triangle GPI$, $\overline{IG} = x' = \overline{IP} \sin \frac{1}{2}(i_1 + i_2)$, hence

$$\frac{x_0}{x'} = \frac{\cos \tfrac{1}{2}(i_1 - i_2)}{\sin \tfrac{1}{2}(i_1 + i_2)}. \tag{8}$$

In the $\triangle$ QPE, $\overline{QP} = y_0 = \overline{EP} \cos \tfrac{1}{2}(i_1 - i_2)$; but, in the $\triangle$ HEP, $\overline{HP} = y' = \overline{EP} \sin \tfrac{1}{2}(i_1 + i_2)$, hence

$$\frac{y_0}{y'} = \frac{\cos \tfrac{1}{2}(i_1 - i_2)}{\sin \tfrac{1}{2}(i_1 + i_2)}. \tag{9}$$

Solving equations (6) and (7) for x' and y' we find

$$x' = \tfrac{1}{2} b [(1 + 2 \tan \tau) \sin \tfrac{1}{2}(i_1 + i_2) + \cos \tfrac{1}{2}(i_1 + i_2)] \sin \tfrac{1}{2}(i_1 + i_2). \tag{10}$$

$$y' = \tfrac{1}{2} b [(1 + 2 \tan \tau) \cos \tfrac{1}{2}(i_1 + i_2) - \sin \tfrac{1}{2}(i_1 + i_2)] \sin \tfrac{1}{2}(i_1 + i_2). \tag{11}$$

The results obtained by substituting these expressions in equations (8) and (9) are

$$x_0 = \tfrac{1}{2} b [(1 + 2 \tan \tau) \sin \tfrac{1}{2}(i_1 + i_2) + \cos \tfrac{1}{2}(i_1 + i_2)] \cos \tfrac{1}{2}(i_1 - i_2). \tag{12}$$

$$y_0 = \tfrac{1}{2} b [(1 + 2 \tan \tau) \cos \tfrac{1}{2}(i_1 + i_2) - \sin \tfrac{1}{2}(i_1 + i_2)] \cos \tfrac{1}{2}(i_1 - i_2). \tag{13}$$

Formulæ (10), (11), (12) and (13) are perfectly general and, in so far as they determine uniquely x_0, y_0, x' and y' when b, τ, i_1 and i_2 are assigned, they are complete. It may be remarked, in passing, that formulæ (10) and (11) show that the position of the axis of rotation with respect to the prism itself is dependent not upon the separate angles i_1 and i_2 but upon the mean angular position $\tfrac{1}{2}(i_1 + i_2)$, whereas x_0 and y_0 involve the interval $i_1 - i_2$ in addition. Moreover, since τ and i_2 are each acute while i_1 can not exceed 90° it follows that x_0 and x' are essentially positive. On the other hand, both y_0 and y' may be simultaneously positive, or zero, or negative. If, for any reason, it should be desirable to have the axis of rotation intersect the optic axis of the telescope this could be accomplished by merely choosing such values for τ, i_1 and i_2 as would satisfy the condition $1 + 2 \tan \tau - \tan \tfrac{1}{2}(i_1 + i_2) = 0$. Equations (8) and (9) show that $x_0 y' - y_0 x' = 0$ and this relation affords a simple check on the results of numerical calculations of the four coördinates involved.

By introducing an auxiliary angle ω which satisfies the relation $\tan \tfrac{1}{2}\omega = 1 + 2 \tan \tau$ formulæ (10) to (13) may be transformed into the following expressions which are convenient for logarithmic computation:

$$x' = A \sin \tfrac{1}{2}(i_1 + i_2), \tag{10'}$$

$$y' = -B \sin \tfrac{1}{2}(i_1 + i_2), \qquad (11')$$

$$x_0 = \quad A \cos \tfrac{1}{2}(i_1 - i_2), \qquad (12')$$

$$y_0 = -B \cos \tfrac{1}{2}(i_1 - i_2). \qquad (13')$$

In these equations

$$A \equiv \frac{b \cos \tfrac{1}{2}(i_1 + i_2 - \omega)}{2 \cos \tfrac{1}{2}\omega} \quad \text{and} \quad B \equiv \frac{b \sin \tfrac{1}{2}(i_1 + i_2 - \omega)}{2 \cos \tfrac{1}{2}\omega}.$$

The condition $\tan \tfrac{1}{2}\omega = 1 + 2 \tan \tau$ requires $\tfrac{1}{2}\omega$ to lie between 45° and 71° 34′ since τ is comprised within the interval 0° to 45°.

When τ and the angles of incidence, I_1 and I_2, corresponding to the ends of the spectral region to be studied are given, equations (10), (11), (12) and (13) are not sufficient for the unique determination of the axis of rotation, since i_1 and i_2 would not be known. Consequently the next step in the analysis is to find a functional dependence of i_1 and i_2 upon I_1 and I_2. (Generally the indices of refraction, n_1 and n_2, for the ends of the spectral interval would be known, but, when τ is also assigned, this is equivalent to having I_1 and I_2 given since $\sin I_1 = n_1 \sin \tau$ and $\sin I_2 = n_2 \sin \tau$.)

Now, for angles of incidence i which are not equal to either i_1 or i_2 the optic axis of the collimator does not meet the incidence face at the center of the latter and therefore the axis of the emergent beam does not coincide with the optic axis of the telescope. Hence we shall so determine i_1 and i_2 as to cause the axis of the emergent beam to depart as little as possible from $\bar{O}X$ during the rotation of the prism from the angular position I_1 to the final position I_2. During this motion the points of incidence, of total reflection and of emergence oscillate along their respective deviating planes in a manner which apparently makes it difficult to see just what path the point of emergence traces out in the plane of the fixed coördinate axes. It is therefore necessary to find an expression for the ordinate of the point of emergence corresponding to any orientation of the prism as characterized by the variable angle of incidence i. Let $\alpha\beta\gamma\delta$ represent the prism in any such position. For the sake of complete generality and to avoid an excessively complicated figure we shall, from now on, make use of the methods of elementary analytical geometry.

The point of emergence, ε, will be found as the intersection of the lines $\beta\alpha$ and $\eta\varepsilon$ where $\eta\varepsilon$ is perpendicular to the line $\lambda\eta$ at the point of internal reflection η. To find the equation of the line $\beta\alpha$ in the normal form we may proceed as follows. The perpendicular, p, dropped from the origin O upon $\beta\alpha$ is parallel to $\delta\alpha$ and hence it makes the angle i with OX. Accordingly the equation of $\beta\alpha$ may be written $x\cos i + y\sin i - p = 0$. Also $\beta\alpha$ is at the distance y' from the point (x_0, y_0) hence $y' = x_0\cos i + y_0\sin i - p$ and therefore the equation of $\beta\alpha$ in final form is

$$x\cos i + y\sin i - (x_0\cos i + y_0\sin i - y') = 0. \qquad (14)$$

In order to find the equation of $\eta\varepsilon$ it will be necessary to know the length of $O\lambda$ which is the intercept on OY of the line $\delta\alpha$, consequently we shall next deduce the equation of $\delta\alpha$. The normal, p', drawn from the fixed origin to $\delta\alpha$ is parallel to $\beta\alpha$ and hence it forms the angle $90° + i$ with OX. Thus, the equation of $\delta\alpha$ may be written $-x\sin i + y\cos i - p' = 0$. Moreover, the distance from the point P to $\delta\alpha$ is equal to $-x'$ hence $-x' = -x_0\sin i + y_0\cos i - p'$. Consequently the equation of $\delta\alpha$ is

$$x\sin i - y\cos i + (x' - x_0\sin i + y_0\cos i) = 0. \qquad (15)$$

Putting $x = 0$ in equation (15) we obtain

$$O\lambda = x'\sec i - x_0\tan i + y_0. \qquad (16)$$

We are now prepared to find the equation of $\eta\varepsilon$. The perpendicular from O to $\eta\varepsilon$ is $O\zeta = \theta\eta = \lambda\eta - \lambda\theta$ and it makes the angle $270° + i - \tau$ with the positive direction of the axis of x.

The equation of $\eta\varepsilon$ may be written $x\sin(i-\tau) - y\cos(i-\tau) - \lambda\eta + \lambda\theta = 0$. We shall accordingly evaluate $\lambda\eta$ and $\lambda\theta$ in terms of $O\lambda$ so as to take advantage of relation (16). In the $\triangle O\theta\lambda$, $\lambda\theta = O\lambda\cos(i-\tau)$ and, furthermore, in the $\triangle \delta\eta\lambda$, $\lambda\eta$: $\sin(45° + \tau) = \delta\lambda : \sin 45° = (b - \lambda\alpha) : \sin 45°$ whence $\lambda\eta = (b - \lambda\alpha)(\sin\tau + \cos\tau)$. But $\lambda\alpha = \mu\nu = O\nu - O\mu$ and, since $O\nu$ is the perpendicular dropped from O upon the line $\beta\alpha$, equation (14) gives $O\nu = x_0\cos i + y_0\sin i - y'$. Finally, in the $\triangle O\mu\lambda$, $O\mu = O\lambda\sin i$ hence, using equation (16), we find $\lambda\alpha = x_0\sec i - x'\tan i - y'$. By making obvious use of this result the relation $\lambda\eta = (\sin\tau + \cos\tau)(b - x_0\sec i + x'\tan i + y')$ is obtained. Once more, by substi-

tuting from equation (16) in $\overline{\lambda\theta} = \overline{O\lambda}\cos(i-\tau)$ the equation of $\overline{\eta\varepsilon}$ is found to be

$$x\sin(i-\tau) - y\cos(i-\tau) - (\sin\tau + \cos\tau)(b - x_0\sec i + x'\tan i + y') + (x'\sec i - x_0\tan i + y_0)\cos(i-\tau) = 0. \quad (17)$$

Lastly, we must solve equations (14) and (17) for y, which is then the required ordinate of the point of emergence, which we shall denote by y_1. After making simple reductions it comes out that

$$y_1 = x_0 + y_0 - x'(\sin i - \cos i) - y'(\sin i + \cos i) - b(1 + \tan\tau)\cos i. \quad (18)$$

Substituting in equation (18) the expressions for x', y', x_0 and y_0, as given respectively by formulæ (10), (11), (12) and (13) and also carrying out elementary trigonometrical reductions which are far too long for publication the following result is obtained

$$y_1 = 2b\{\cos\tfrac{1}{2}(i_1+i_2) + [\sin\tfrac{1}{2}(i_1+i_2) + \cos\tfrac{1}{2}(i_1+i_2)]\tan\tau\}\sin\tfrac{1}{2}(i-i_1)\sin\tfrac{1}{2}(i-i_2). \quad (19)$$

By equating to zero the first derivative of y_1 with respect to i we find that y_1 has a stationary value when $i = \frac{1}{2}(i_1 + i_2)$. The second derivative shows that y_1 always has an algebraic minimum when i assumes this mean value. The minimum of y_1 may be written

$$y_2 = -2b\{\cos\tfrac{1}{2}(i_1+i_2) + [\sin\tfrac{1}{2}(i_1+i_2) + \cos\tfrac{1}{2}(i_1+i_2)]\tan\tau\}\sin^2\tfrac{1}{4}(i_1-i_2). \quad (20)$$

The fact that y_1 attains its minimum when $i = \frac{1}{2}(i_1 + i_2)$ is interesting because this position of the prism is not the one at which the points of incidence, of total reflection, and of emergence experience a reversal of direction of motion along the respective deviating faces. As is easily shown, these turning points are reached when $i = \sin^{-1}[\sin\frac{1}{2}(i_1+i_2) : \cos\frac{1}{2}(i_1-i_2)]$, that is when $i > \frac{1}{2}(i_1+i_2)$.

On the other hand, the simplicity of the geometrical meaning of the condition $i = \frac{1}{2}(i_1 + i_2)$ would naturally lead one to suppose that this result could have been established at once from the geometrical properties of the figure and that therefore the preceding

analysis is unnecessary. However, this supposition is not strengthened by the facts that $x_1 - x_0$ does not involve the mean angle, $\frac{1}{2}(i_1 + i_2)$, symmetrically and that the equation in x and y of the point of emergence is, in general, of the fourth degree with very complicated literal coefficients.

Fig. 3 shows the curve traced out in the plane XOY by the point of emergence ε for the special case of Jena glass S. 228 where $b = 44.7$ cm. and $\tau = 30°$. (See also Table II, first row.) Points 6

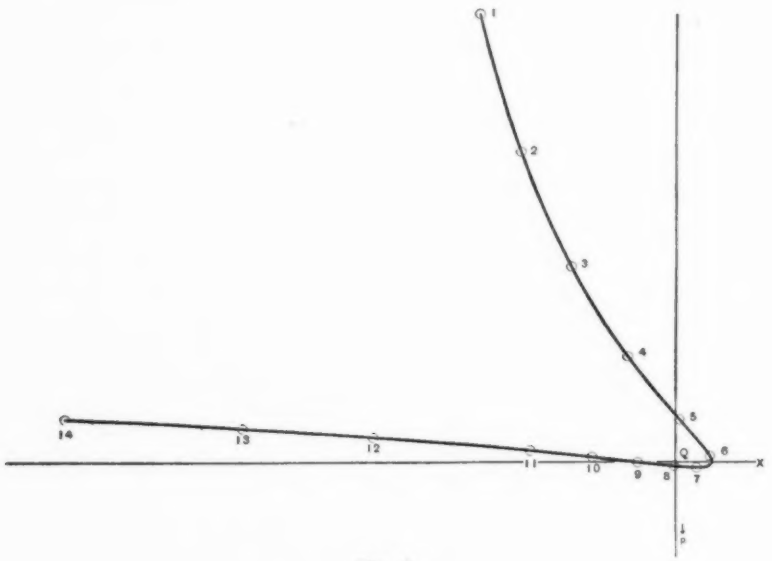


Fig. 3.

and 10 are at the ends of the arc which pertains to the axis of rotation through P and hence to the spectral region from A' to G'. The angles of incidence corresponding to the points 1 to 14 are respectively 45°, 50°, 55°, 60°, 65°, 70° 5′ 25″, 74° 26′ 40″, 76°, 77° 31′ 22″, 78° 47′ 54″, 80°, 82°, 83° and 84°.

It follows at once from equation (19) that y_1 is symmetrical with respect to $\frac{1}{2}(i_1 + i_2)$ in the sense that this ordinate has the same value for any two values of i such that the one exceeds $\frac{1}{2}(i_1 + i_2)$ by the same amount as the other falls short of this mean angle. (This is also illustrated by Table II.) Since such is the case, it is now a simple matter to impose upon i_1 and i_2 the condition that y_1 shall

have the least possible arithmetical maximum within, and at the limits of, the angular interval from I_1 to I_2. For this condition to hold it is both necessary and sufficient that the numerical value of y_1 at its minimum shall be equal to the values of y_1 at the two boundaries of the interval. The equality of the values of y_1 at the ends of the interval leads to

$$i_1 + i_2 = I_1 + I_2. \tag{21}$$

Again, as stated above, $|y_2|$ must equal y_1 when $i = I_1$ (or $i = I_2$), hence, referring to equations (19) and (20), we see that

$$\sin \tfrac{1}{2}(I_1 - i_1) \sin \tfrac{1}{2}(I_1 - i_2) = \sin^2 \tfrac{1}{4}(i_1 - i_2)$$

and therefore

$$\cos \tfrac{1}{2}(i_1 - i_2) - \cos [I_1 - \tfrac{1}{2}(i_1 + i_2)] = 2 \sin^2 \tfrac{1}{4}(i_1 - i_2).$$

Substituting for $i_1 + i_2$ the sum given by equation (21) and making simple transformations we find

$$i_1 - i_2 = 4 \sin^{-1} \left[\frac{1}{\sqrt{2}} \sin \tfrac{1}{4}(I_1 - I_2) \right]. \tag{22}$$

Finally, by solving equations (21) and (22) for i_1 and i_2 it is found that

$$\begin{aligned} i_1 &= \tfrac{1}{2}(I_1 + I_2) + 2 \sin^{-1} \left[\frac{1}{\sqrt{2}} \sin \tfrac{1}{4}(I_1 - I_2) \right], \\ i_2 &= \tfrac{1}{2}(I_1 + I_2) - 2 \sin^{-1} \left[\frac{1}{\sqrt{2}} \sin \tfrac{1}{4}(I_1 - I_2) \right]. \end{aligned} \tag{23}$$

As has been tacitly assumed, the preceding equations require that $I_1 > i_1 > i_2 > I_2$.

The solution of the main problem is now complete since the combination of equations (21) and (22) with (12), (13) and (20) leads to the final explicit formulæ,

$$\begin{aligned} x_0 = \tfrac{1}{2}b[&\cos \tfrac{1}{2}(I_1 + I_2) + \sin \tfrac{1}{2}(I_1 + I_2) \\ &+ 2 \sin \tfrac{1}{2}(I_1 + I_2) \tan \tau] \cos^2 \tfrac{1}{4}(I_1 - I_2), \end{aligned} \tag{24}$$

$$\begin{aligned} y_0 = \tfrac{1}{2}b[&\cos \tfrac{1}{2}(I_1 + I_2) - \sin \tfrac{1}{2}(I_1 + I_2) \\ &+ 2 \cos \tfrac{1}{2}(I_1 + I_2) \tan \tau] \cos^2 \tfrac{1}{4}(I_1 - I_2), \end{aligned} \tag{25}$$

$$\begin{aligned} y_2 = -b\{&\cos \tfrac{1}{2}(I_1 + I_2) + [\cos \tfrac{1}{2}(I_1 + I_2) \\ &+ \sin \tfrac{1}{2}(I_1 + I_2)] \tan \tau\} \sin^2 \tfrac{1}{4}(I_1 - I_2). \end{aligned} \tag{26}$$

In virtue of the fact that $i_1 + i_2 = I_1 + I_2$, formulæ (10) and (11) remain unchanged. Since $\tan \tau < 1$ it follows that $x' < 1.58b \sin \frac{1}{2}(I_1 + I_2)$, $x_0 < 1.58b$ and $|y_2| < 2.24b \sin^2 \frac{1}{4}(I_1 - I_2)$. In general, $\sin^2 \frac{1}{4}(I_1 - I_2)$ is a small quantity and hence y_2 is a small per cent. of b. Obviously when $\frac{1}{4}(I_1 - I_2)$ is very small $\sin^2 \frac{1}{4}(I_1 - I_2)$ and $\cos^2 \frac{1}{4}(I_1 - I_2)$ may be replaced by $1.9039 \times 10^{-5}(I_1^\circ - I_2^\circ)^2$ and by 1 respectively.

The auxiliary quantities ω, A and B which occur in equations (10′) to (13′) do not change their values, because of the condition $i_1 + i_2 = I_1 + I_2$, and therefore they may be used to reduce formulæ (24), (25) and (26) to the following expressions which are especially adapted to logarithmic computation.

$$x' = A \sin \tfrac{1}{2}(I_1 + I_2), \qquad (10')$$

$$y' = -B \sin \tfrac{1}{2}(I_1 + I_2), \qquad (11')$$

$$x_0 = A \cos^2 \tfrac{1}{4}(I_1 - I_2), \qquad (24')$$

$$y_0 = -B \cos^2 \tfrac{1}{4}(I_1 - I_2), \qquad (25')$$

$$y_2 = -\frac{b \tan \tau \cos \left[\frac{1}{2}(I_1 + I_2) - (\frac{1}{2}\omega - 45^\circ)\right] \sin^2 \frac{1}{4}(I_1 - I_2)}{\sin (\frac{1}{2}\omega - 45^\circ)}. \qquad (26')$$

Some practical illustrations are given in Table II., in which the linear unit is the centimeter. The numbers in the fifth row correspond very closely to the prism of a spectroscope made by Hilger. For this particular piece of apparatus $x' = 4.46$ cm., $y' = 0.60$ cm., $x_0 = 5.08$ cm. and $y_0 = 0.44$ cm.

Commencing with the paragraph in which the methods of analytical geometry were introduced, some of the succeeding results apply strictly only to cases where the incident beam is sufficiently narrow to avoid having a lateral portion of the beam cut off by the vertical edges of the incidence face when $i \lessgtr I_1$. In other words, the final formulæ involve implicitly the condition that the entire beam may be continuously represented by its axis during the rotation of the prism from the angular position I_1 to the position I_2. However, another extreme case is met with in practice. It arises when the aperture of the collimator is large enough to completely fill the incidence face with light when $i \gtrless I_2$. For example, the

TABLE II.

Jena No.	b	τ	$\frac{1}{2}\omega$	n_1	n_2	I_1	I_2	x'	y'	x_0	y_0	$-y_2$	$-y_4$
S. 228	4	30° 0′ 0″	65° 6′ 14″	1.9619	1.8805	78° 47′ 54″	70° 5′ 25″	4.516	−0.7428	4.681	−0.7700	0.00565	−0.00111
O. 113	4	30 0 0	65 6 14	1.7721	1.7218	62 22 56	59 25 12	4.140	+0.3042	4.737	+0.3481	0.000850	+0.0000582
O. 102	4	30 0 0	65 6 14	1.6744	1.6374	56 50 48	54 57 14	3.883	+0.6292	4.689	+0.7598	0.000372	+0.0000518
O. 198	4	30 29 57.5	65 20 20	1.8181	1.7610	67 19 41	63 20 59	4.356	0.0000	4.792	0.0000	0.00144	0.0000
O. 469	3.73	32 17 10	66 9 58	1.6756	1.6382	63 30 41	61 2 59	4.076	+0.2768	4.604	+0.3127	0.000495	+0.0000361

latest form of Hilger's constant deviation prism spectroscope, having an index of refraction of about 1.65 for mean D-light, is constructed on this principle. When this condition is fulfilled the width of the effective incident beam equals the projection of the incidence face on any plane perpendicular to the optic axis of the collimator. Hence, the point where the axis of the effective incident beam meets the incidence face is the center of this face for all orientations of the prism within the given angular interval. Therefore, the axis of the emergent beam continually passes through the center of the emergence face. Since the last named point is constrained to move on the circumference of a circle having P as center and PE as radius it is a very simple matter to impose upon i_1 and i_2 the condition that, on the whole, the center of the emergence face shall depart from the optic axis of the telescope by as small an amount as possible within and at the boundaries of the region from I_1 to I_2. Accordingly we shall omit the mathematical details and merely state the final results.

If y_3 denotes the ordinate of the center of the emergence face corresponding to the position i then

$$y_3 = \frac{2y_0}{\cos \frac{1}{2}(i_1 - i_2)} \cdot \sin \tfrac{1}{2}(i - i_1) \sin \tfrac{1}{2}(i - i_2). \qquad (27)$$

This equation is the precise analogue of formula (19) and, since it involves the variable angle i in identically the same manner as (19), relations (21) and (22) again apply and lead to the final ordinate

$$y_4 = -y_0 \tan^2 \tfrac{1}{4}(I_1 - I_2). \qquad (28)$$

This expression is, of course, the analogue of formula (26) or (26′). Equation (28) shows that y_4 vanishes simultaneously with y_0 and, as has already been shown, y_0 equals zero when

$$1 + 2 \tan \tau - \tan \tfrac{1}{2}(I_1 + I_2) = 0.$$

Thus, it is possible, in general, to so choose τ as to cause no displacement of the axis of the emergent beam with respect to the optic axis of the telescope during the rotation of the prism. In this connection, attention may be called to the fact that when the indices of refraction n_1 and n_2, corresponding to the ends of the interval of rotation are given, it is not a simple matter to calculate τ so as to

make $y' = y_0 = y_4 = 0$ because I_1 and I_2 involve the unknown quantity τ by virtue of the relations $\sin I_1 = n_1 \sin \tau$ and $\sin I_2 = n_2 \sin \tau$. In fact, the rationalized equation for $\tan \tau \equiv t$ is the following inconvenient polynomial of the sixth degree, namely:

$$8(n_1 - n_2)^2 t^6 + 16(n_1 - n_2)^2 t^5 + 2[(n_1 + n_2)^2 + 7(n_1 - n_2)^2 - 4]t^4$$
$$+ 2[(n_1 + n_2)^2 + 3(n_1 - n_2)^2 - 4]t^3$$
$$+ [(n_1 + n_2)^2 + (n_1 - n_2)^2 - 10]t^2 - 8t - 2 = 0.$$

The simpler plan to follow, in any numerical case, is to use the so-called method of successive trial and error. In the latter way the numbers in the fourth row of Table II. were calculated. The last column of this table contains the values of y_4 as obtained from formula (28).

In conclusion it may not be superfluous to state *en résumé* that what we have done was first, to generalize the usual statements relative to Pellin and Broca's type of prism; second, to have demonstrated formulæ (10), (11), (12) and (13) so that i_1 and i_2 may be subjected by the designer to any conditions whatsoever compatible with the nature of the problem; and third, to have subjected i_1 and i_2 and the various formulæ involving these angles to a set of conditions which, in the two most important and extreme cases, cause the axis of the emergent beam to suffer very small displacements from the optic axis of the telescope. The numbers in the columns of Table II. headed $-y_2$ and $-y_4$ show how well conditions (21) and (22) solve the main problem proposed for solution.

SLOANE PHYSICAL LABORATORY,
YALE UNIVERSITY, April, 1909.

THE ELECTROLYTIC DETECTOR, STUDIED WITH THE AID OF AN OSCILLOGRAPH.

BY GEORGE W. PIERCE.

INTRODUCTION.

WHILE engaged in a series of experiments [1] on certain crystal rectifiers for alternating electric currents — these rectifiers also serving as detectors for electric waves — the writer made use of a sensitive form of Braun's cathode-tube oscillograph. The same apparatus is here applied to the study of the electrolytic detector. The purpose of the investigation is, first, to attempt to contribute to the understanding of the mode of operation of the electrolytic detector, and, second, to find out whether the experiment with the electrolytic detector will throw any light on the crystal detectors which, like the electrolytic detector, are rectifiers for small alternating currents.

Method of Employing the Detector in the Reception of Electric Waves. — The electrolytic detector for electric waves, as described by Fessenden [2] and shortly after by Schloemilch,[3] consists of a cell containing an electrolyte and having one electrode of very small area, usually in the form of an extremely fine wire of platinum, and as the other electrode a larger area of platinum or some other metal. When used in wireless telegraphy the two electrodes are connected in a circuit upon which the electric oscillations are impressed, so that the rapidly oscillating electric currents in the circuit are made to traverse the cell of the detector. An example of a simple form of receiving circuit, with the detector connected in the antenna, is shown at *MDG* of Fig. 1. A local circuit *TED*, through the detector, contains a telephone receiver *T* and an adjustable source of E.M.F., which is used to polarize the detector

[1] Pierce, Proc. Am. Acad., 44, pp. 317–349, March, 1909; PHYS. REV., 23, pp. 153–187, March, 1909.

[2] Fessenden, U. S. Patent No. 727,331, May 5, 1903.

[3] Schloemilch, E. T. Z., 24, p. 959, 1903.

by sending through it and the telephone a small direct current. Under the action of the electric oscillations through the detector the current in the telephone receiver is modified so as to produce a sound in the telephone with a period determined by the train fre-

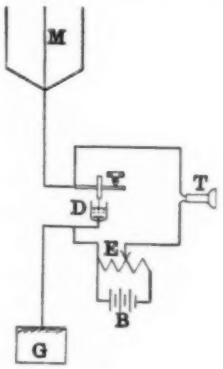


Fig. 1.

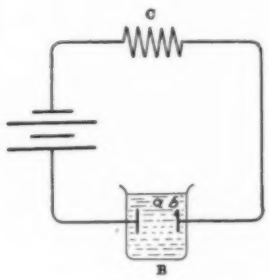


Fig. 2.

quency of the incident electric waves. The action is localized at the contact of the fine wire with the electrolyte.

The Electrolytic Detector as a Rectifier. — That an electrolytic cell with one of the electrodes small, when suitably polarized with a direct current, is a rectifier for alternating currents was first shown by Pupin[1] before such a cell came into commercial use as a detector for electric waves. The following account of Pupin's rectifier is translated from the "Jahrbuch der Elektrochemie," Vol. 6, p. 35, 1899: "In Fig. 3" (here reproduced as Fig. 2) "*A* is a battery, *B* an electrolytic cell with the platinum electrodes *a* and *b* and acidulated water. If the polarization of the cell *B* is as great as the E.M.F. of *A*, no current flows in the circuit. If one allows an alternating current to act upon the circuit *ABC*, the circuit contains resistance, self inductance, and a capacity localized in the plates *a* and *b*. The cell *B* acts, however, as a condenser only so long as the potential difference of the plates *a* and *b* is smaller than the decomposition voltage. If this value is exceeded, a current goes through the circuit. If the alternating current, for example,

[1] Pupin, Electrical World, 34, p. 743, 1899; Zeitsch. f. Elektrochemie, 6, p. 349, 1899; Jahrbuch. d. Elektrochemie, 6, p. 35, 1899; Bul. Am. Phys. Soc., 1, p. 21, 1900.

has an amplitude that is twice as great as the E.M.F. of A, in case the phase has the same direction as A a current flows in the circuit, *e. g.*, in the direction BC; when the phase is oppositely directed, the condenser B sends a current in the opposite direction. This last can be diminished by making the capacity of B very small. If, for example, the area of one of the electrodes is only one square millimeter, one may easily rectify alternating currents with a frequency of 1,000 per second; with greater frequency the electrode must naturally be made still smaller. It is best to employ a platinum wire sealed into glass — the wire being cut off immediately at the end of the glass. The author succeeded in rectifying electric oscillations of Hertzian frequency, and producing electrolytic effects with them; the wire for this purpose was .025 mm. in diameter."

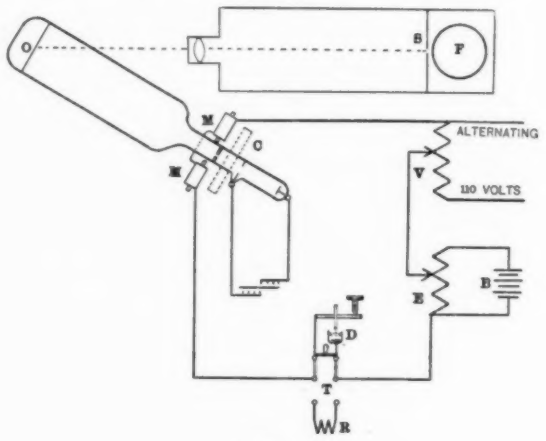


Fig. 3.

This quotation is introduced to show that Pupin had employed the electrolytic detector as a rectifier for electric waves of Hertzian frequency, and that he had a well-defined explanation of the processes occurring in the rectifier. The present experiment falls into close agreement with Pupin's explanation of the phenomenon.

Further study of the electrolytic detector, as a rectifier, has been made by Armagnat,[1] and Austin.[2] Armagnat has taken current-

[1] Armagnat, Bul. soc. française, session of April, 1906, p. 205; Journal de Physique, 5, p. 748, 1906.

[2] Austin, Bul. Bureau of Standards, 2, p. 261, 1906.

voltage curves of the detector with a galvanometer and a source of steady voltage, and by arguments similar to those employed by Pupin (but without reference to the polarization capacity of the electrodes) has shown more in detail how electrolytic polarization accounts for the rectification of alternating currents by the detector. Austin, also using a galvanometer as the indicating instrument, studied the effect of alternating currents of slow frequency and also the effects of electric waves, and came to the opinion that heat, chemical action, rectification and electrostatic attraction across the gas film might have a part in the explanation of the phenomenon.

In the present experiments the current through the detector under the action of an alternating E.M.F., superposed on a polarizing current, is determined by means of an oscillograph. The application of the oscillograph to the problem gives the instantaneous values of the current through the detector, and permits an examination of the wave form of the rectified cycle.

Oscillographic Apparatus.

Circuits Employed with the Detector in Taking the Oscillogram. — The electrolytic detector used in these experiments made use of a platinum point, .005 mm. in diameter, dipping into 20 per cent. nitric acid, and was adjusted to high sensitiveness as an electric wave detector immediately before taking the oscillograms. A diagram of the circuits employed in the experiment, together with a sketch of the oscillographic apparatus, is shown in Fig. 3. The detector is at D, and is connected in series with the deflecting coils MM of the oscillograph and with the variable sources of voltage V and E. The voltage V is taken from a potentiometer connected with the 60-cycle alternating mains of the laboratory. E is an adjustable steady voltage taken from a battery. The voltage at E could be reversed. By opening the switch at S the electrolytic detector could be disconnected from the circuit, and by throwing this switch to the right an ohmic resistance could be substituted for the detector.

The Braun's Tube Oscillograph. — The oscillographic apparatus was essentially the same as that employed in the experiments on

crystal rectifiers (Part II.).[1] In the present experiments the high potential through the tube was produced by a Holtz influence machine, driven by an electric motor, whereas in the previous experiments Professor Trowbridge's high potential battery was used. The moving plates of the influence machine were of plain glass and did not carry any sectors or discs such as are used on the most common types of Toeppler-Holtz machines. This is important, since such sectors or discs produce a pulsewise discharge through the cathode tube, which results in a broadening of the cathode spot when it is deflected. The present machine in which the electricity was taken directly from the moving glass plates was very constant and gave cathode rays of great homogeneity, so that the luminescent spot produced at *O* on the screen of the tube remained sharply defined however far it was deflected.

The photograph of the moving luminescent spot was taken on a film carried by a rotating drum *F*, which made 20 revolutions per second about a horizontal axis. The drum was driven by a synchronous motor operating on the 60-cycle alternating mains of the laboratory, from which the alternating current sent through the rectifier was also taken. The synchronism of the drum with the deflections of the spot was so perfect that very long exposures could be made without any failure of perfect superposition, and without any appreciable fogging of the film.

The deflecting electromagnets *MM* had a combined resistance of 436 ohms, and were provided with soft iron cores about 6 mm. in diameter. With the small current employed the iron showed no appreciable hysteresis, and the deflections of the light spot were proportional to the currents. In oscillogram No. 1 (Plate I.) a deflection of .85 cm. on the film was obtained with a current of 1 milliampere. In taking the pictures, except No. 1, a Ryan focusing coil was placed about the tube so as to give a longitudinal field, which could be adjusted to bring the luminescent spot to have a circular area about .5 millimeter in diameter. This focusing device diminished the sensitiveness of the apparatus to about one third so that 1 milliampere gave a deflection of .28 cm. on the film, but the employment of the Ryan coil is a great advantage in producing a narrow line for the curves.

[1] G. W. P., *l. c.*

In taking the oscillograms the following steps were taken: The drum carrying the film was set rotating. The high potential current was started in the tube. The chosen value of the polarizing current was applied to the circuit and was read on a direct-current milliammeter. The alternating current was superposed on the circuit, and by adjustment of the potentiometer at V the voltage of this alternating current was given any desired value.

The Exposures.

After the preliminary adjustment of the direct and alternating currents through the detector, four exposures were made on each picture, while the film was being carried around continuously by the synchronously driven drum.

Axis of Zero Current. — This is the lower straight line across the pictures, and was obtained by an exposure of 20 seconds taken with the circuit open.

Axis of Polarizing Current. — This is the upper straight line across the picture, and was obtained with the detector in circuit and traversed by the polarizing current. The exposure was 20 seconds. In oscillogram No. 1 this axis is not apparent because on account of the small value of the polarized current employed it falls into coincidence with the axis of zero current.

The Rectified Cycle. — This cycle may be identified in the oscillograms as a positive[1] loop for a half-period, followed by a nearly straight portion lying along the axis of zero current for a part of a half cycle, and going over into the positive loop through an intermediate "building up" segment. This cycle (exposure of 60 sec.) was taken with the detector in circuit, with the alternating E.M.F. applied to the circuit, and with the polarizing current also flowing.

The Voltage-Phase Cycle. — This the sine-curve of the pictures, and was taken in order to obtain the E.M.F. immediately about the detector.[2] A similar curve was made use of in the writer's

[1] In describing the oscillograms, values *above* the axis of zero current are called positive; values below this axis are called negative.

[2] The ordinary method, which would be to take the leads from the two sides of the detector through a high resistance to the oscillograph, could not be used because the oscillograph was working at the limit of its sensitiveness on the full voltage without the added resistance.

experiments on crystal rectifiers and is there discussed. In the present experiments, because of the employment of the polarizing current with the rectifier, a question arises as to the appropriate method of taking this cycle. Two different methods were tried, either of which by proper elimination of the constants of the oscillographic apparatus, will give the desired result. The method yielding simplest results for the voltage-phase cycle is the following: After the exposure for the rectified cycle had been made, the alternating voltage was left unchanged, and a resistance was substituted for the rectifier. A double adjustment of the substituted resistance and the direct voltage was made by successive approximations until the result was attained that (1) the direct voltage alone gave through the substituted resistance a current equal to that used in polarizing the rectifier and (2) the alternating voltage superposed on this direct current gave a deflection of the luminescent spot to a point coincident with the maximum point attained with the rectifier in the circuit. This means that the voltage-phase cycle was taken with the axis of polarizing current as axis, and with amplitude equal to the maximum amplitude of the rectified cycle. This method was employed in oscillograms 1, 2 and 5.

TABLE I.

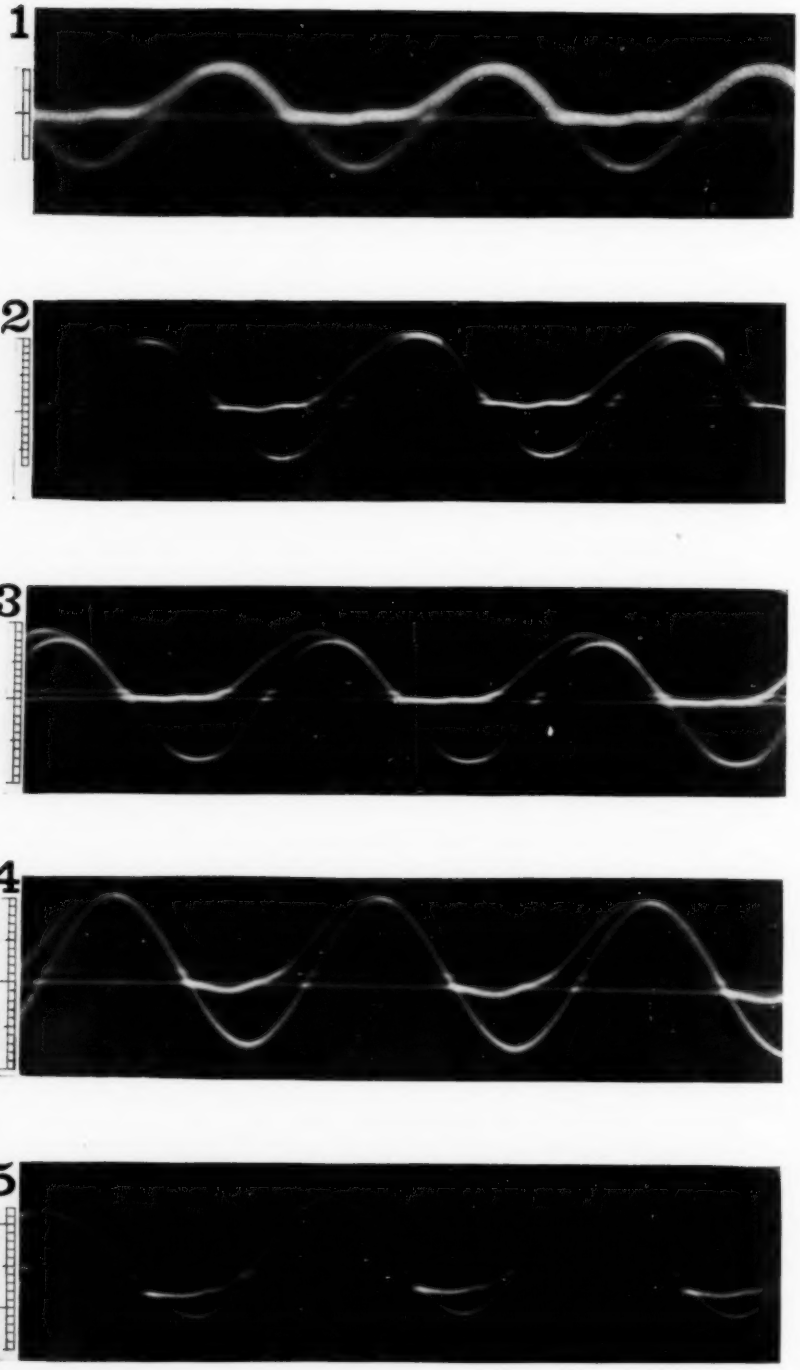
Tabular Description of the Oscillographic Records.

No.	Polarizing Direct Current in Milliamperes.	Polarizing E.M.F. in Volts.	R.M.S. Volts A.C.	Maximum Positive Current through Detector in Milliamperes.	Equivalent Resistance in Ohms.
1[1]	Less than .1	1.45	2.09	2.37	440 [1]
2	1.0	5.5	4.00	9.6	70
3[2]	1.2	5.5	4.00	9.6	00 [2]
4[2]	1.4	Not measured.	5.00	10.0	00 [2]
5	2.2	"	5.00	11.0	150

The second method of taking the voltage-phase cycle was as follows: The polarizing voltage was reduced to zero, the detector was short-circuited, and an alternating voltage equal to that used with

[1] It should be noticed that the sensitiveness of the oscillograph when No. 1 was taken was three times as great as when the other oscillograms of the plate were taken

[2] The voltage-phase cycle of oscillograms 3 and 4 were taken with the polarizing current omitted, so that they have the axis of no current as axis of the cycle.



G. W. PIERCE.

The Electrolytic Detector.

the detector was applied to the circuit. This method was employed in oscillograms 3 and 4.

Coördinates of the Oscillographic Curves. — In taking all of the curves of the oscillograms, the motion of the light spot over the film is from left to right; the time coördinate is, therefore, the abscissa of the curves and is drawn as usual from left to right. The current coördinate is given in the scale drawn in ink at the left hand margin of each picture — one division being one milliampere.

Discussion of the Oscillograms of Plate I.

The oscillograms shown in Plate I. are reproductions of positives printed from the films carried by the rotating drum. The reproduction is one third the size of the original. They were taken with a 60 cycle alternating current applied to the circuit containing the electrolytic detector. The several curves shown in the plate were obtained with different polarizing currents superposed on the circuit. Table I. contains a tabulation of the polarizing current and voltage, the applied alternating voltage, the maximum current through the detector and the substituted resistance employed in taking the voltage curve.

Point Anode or Cathode — the Large Loop in the Direction of the Polarizing Current. — Some of the oscillograms were taken with the polarizing current from the point to the electrolyte and some with the polarizing current in the opposite direction. Although the values of the polarizing voltage required to produce a given polarizing current were different in the two cases the general characteristics of the cycle were the same. A reversal of the polarizing current reversed the rectified current, and whether the polarizing current was from the point to electrolyte or in the opposite direction the large loop of the rectified cycle (always oscillographed positively) was obtained when the alternating current was flowing in the same direction as the polarizing current.

The Form of the Rectified Cycle. — The cycle obtained with the rectifier in the circuit has the same general form in all the pictures. When the current, having traversed the positive loop, comes to the axis of zero current, it follows along this axis for a short way, then takes a small negative dip, becomes positive again, follows

along just above the axis of zero current for a short time, and then rises along a transition curve to the positive loop.

Calculations Concerning the Form of the Cycle. — The rectified cycle, when examined by comparison with the voltage-phase cycle, makes a misleading impression unless one takes carefully into account the condition under which the curves are obtained. One must bear in mind that the form of the current through any rectifier is not determined by the rectifier alone, but is a function also of the constants of the circuits employed with the rectifier. In the present experiments, the deflecting coils of the oscillographic apparatus possessed appreciable self inductance and resistance, and these factors must be taken into account. It is proposed, therefore, to examine the problem in the light of the elementary theory of alternating currents.

The examination will be confined to oscillogram No. 2. The data for this curve are contained in Table I. The polarizing cur-

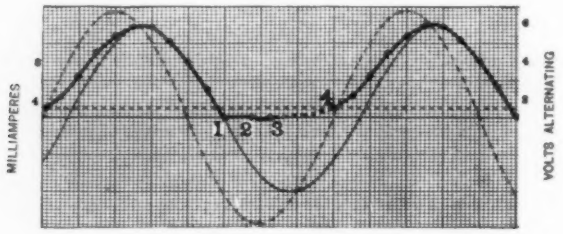


Fig. 4.

rent had a value of .001 ampere, obtained by impressing a steady voltage of 5.5 volts on the circuit containing the detector and the deflecting coils. The impressed alternating voltage had a R.M.S. value of 4 volts, and therefore a maximum value of $4\sqrt{2} = 5.66$ volts. The equation of this voltage is

$$(1) \qquad e = 5.66 \sin \omega t,$$

which is plotted as the dotted sine-curve of Fig. 4. The ordinate scale for this curve, in volts, is at the right of the diagram.

The voltage-phase cycle taken with this alternating E.M.F. and with a direct current of .001 ampere impressed upon the circuit, which consists of the coils of the oscillograph and a resistance of 70 ohms substituted for the rectifier, has the equation

$$(2) \qquad i_1 = \frac{5.66}{\sqrt{R^2 + L^2\omega^2}} \sin\left(\omega t - \tan^{-1}\frac{L\omega}{R}\right) + 1 \times 10^{-3}.$$

R is made up of the resistance of the coil, 436 ohms, plus the added resistance of 70 ohms, making 506 ohms. The maximum value of i_1, taken from the oscillogram, is 9.8×10^{-3} amperes. The values of R and $i_{1(\max)}$ substituted in equation (2) gives

$$9.8 \times 10^{-3} = \frac{5.66}{\sqrt{506^2 + L_2\omega}} + 1 \times 10^{-3};$$

whence

$$(3) \qquad L\omega = 397,$$

and

$$\varphi_1 = \tan^{-1}\frac{397}{506} = 38.1°,$$

where φ_1 is the angle of lag of the voltage-phase cycle behind the voltage e of equation (1). The value of i_1 is now completely known, and is plotted as the *continuous* sine-curve of Fig. 4. This curve is the voltage-phase cycle of the oscillogram No. 2.

Let us next compute some points of the rectified cycle; namely, the points indicated by circles and numbered 1, 2, 3 and 4 on the diagram of Fig. 4. Let us at first assume that the rectified cycle and the voltage phase cycle come to the axis of zero current together. This gives us the point 1 as a starting point. Then the value of the current at the points 2 and 3 is also zero, because the impressed voltage is zero, being 5.5 volts (the polarizing E.M.F. externally applied) and -5.5 volts (the corresponding ordinate of the dotted sine-curve). Thus the externally applied voltage is zero. Also, since the current is practically zero and the rate of change of current is practically zero in the neighborhood of the points 3 and 4, the internal E.M.F. of the circuit due to resistance and inductance is zero. This gives the total voltage as zero and the resultant current zero at these points. The point 4 has the abscissa 2π, therefore the external alternating voltage (dotted curve) is zero; whence the current is equal to the polarizing current, and the point is on the axis of polarizing current. (This assumes that the current between points 3 and 4 is small and its rate of change is small, which is borne out by the fact that the current in this region has to remain between the two axes.)

The points 2, 3 and 4 are thus located by calculation. The march of the current intermediate between the points 3 and 4 is difficult to determine, on account of the unknown way in which the current grows during the polarization of the electrode. I have, therefore, indicated the current in this interval by a dotted line. We know, however, that between the points 2 and 3 there must be a tendency to a small negative maximum, because in this interval the negative value of the alternating voltage, taken from the dotted sine-curve, is larger than the positive polarizing voltage (5.5 volts). This small negative maximum is indicated on the diagram.

Having obtained the location of the four points 1, 2, 3 and 4, and of the negative loop between 3 and 4, let us next consider the large loop of the rectified cycle. In this loop the alternating voltage and the polarizing voltage aid each other, so as to produce large values of the current. With these large values of the current the resistance and the inductance of the coils come to play an important part, and the current tends to traverse an arc of a sine curve. However, since the current has been nearly zero for a large part of a half period, the "building-up" terms of the current equation are of importance. Let us attempt to obtain an approximate equation for this part of the cycle. To do this we need to keep in mind the experimental fact that it takes several volts to overcome the E.M.F. of polarization of the lectrolytic cell with small electrode, but after this polarization has been overcome, the application of additional voltage produces additional current proportional to the additional voltage. This fact is illustrated by the current-voltage curve of Fig. 5, taken from the work was obtained by Armagnat by applying various values of steady of Armagnat.[1] This curve voltage and measuring the current through the detector with a galvanometer. If we assume that a similar relation of current to voltage obtains when the voltage is rapidly changing with the

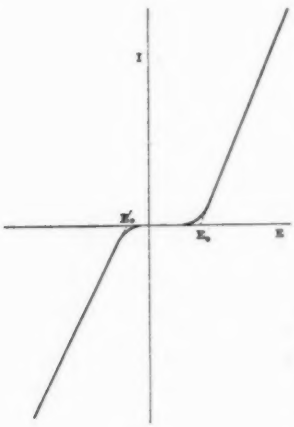


Fig. 5.

[1] *L. c.*

time, we shall be able to compute approximately the upper loop of the cycle. When the current in the positive direction in our experiments has reached the axis of polarizing current it is on the straight part of the curve of Fig. 5, so that above this axis the resultant current is made up of .001 ampere due to the polarizing voltage plus the current under the action of the alternating voltage — this added current being obtained by assuming that the resistance of the detector is constant. This assumption gives for the current of the upper loop of the rectified cycle, the following equation:

$$(4) \qquad i_2 = \frac{5.66 \sin\left(\omega t_1 - \tan^{-1}\frac{L\omega}{R}\right)}{\sqrt{R^2 + L^2\omega^2}} + .001 + c\varepsilon^{-\frac{Rt}{L}},$$

in which t_1 is measured from the instant at which ωt becomes 2π, 4π, 6π, etc. $L\omega$ has the value given in equation (3) above, and because of the equality of the positive amplitude of the rectified cycle and that of the voltage-phase cycle, we know that R has the value it had above; namely, 506 ohms. Putting these values in equation (4) and remembering that when $\omega t = 2\pi$, $i_2 = .001$, and $t_1 = 0$, we may determine the constant c. The value obtained is $c = .0054$ ampere. By a comparison of equations (4) and (2) we have

$$(5) \qquad i_2 = i_1 + .0054\varepsilon^{-\frac{Rt}{L}}.$$

By computing the values of the exponential term and adding them to the corresponding values of i_1 we obtain the values of i_2. These are plotted as the positive loop in Fig. 4, which, together with the points 1, 2, 3 and 4, make up the complete computed curve for the current through the rectifier.

In justification of our assumption of the point 1 as a starting point it should be noted that, since the exponential becomes inappreciable in less than a half period, i_2 becomes equal to i_1 and the two currents approach the axis of zero current together.

Comparison of the Computed Cycle with the Cycle of the Oscillogram No. 2. — The scale used in plotting the computed curves is not identical with that of the original oscillogram. To show the agreement of the form of the computed curve of Fig. 4 with oscillogram No. 2, a tracing of the original oscillogram is drawn in Fig.

6. The computed values of the rectified cycle, reduced to the same scale, are also plotted as the circles of Fig. 6.

The computed points agree well with the oscillogram in the location of the negative maximum, the position of which with respect to the positive maximum is seen to be entirely due to the inductance and resistance of the circuit. The computed points also fall well into agreement with the oscillogram on the right-hand slope of the

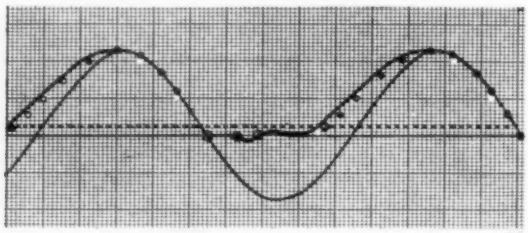


Fig. 6.

positive loop, but on the left-hand slope there is a considerable departure. On this slope the computed values of the current are all too low. This might have been expected because the "building up" had really begun earlier than the time assumed; that is, before the cycle had reached the point 4; and the computation in this segment is to be regarded only as a rough approximation.

Evidence of Polarization Capacity. — On oscillograms 1, 2 and 3 there is a small positive rise of the photographic curves in the region to the immediate right of the negative maximum. This rise is more striking in the original photographs than in the reproductions; and, though small, it deserves attention, because the occurrence of this small positive maximum is evidence of the existence for about 1/1,500 of a second of a positive E.M.F. greater than the E.M.F. immediately following. Now in this part of the cycle the externally applied E.M.F. is greater following the rise than during the rise; therefore the rise indicates the existence of a positive E.M.F. in the circuit itself. This is capable of the following explanation in terms of the theory of polarization. After the prevalent external E.M.F. has been in a negative direction and has returned to zero, the polarization tension which has been opposing the negative current at the electrode continues to exist for a short time and produces a positive

current. This action, resembling that of a capacity, is familiarly known as the polarization capacity of the electrode. By the existence of the small positive maximum near the axis of the cycle, the oscillogram shows that the polarization capacity of the electrode is not entirely negligible. Evidence of the existence of this polarization capacity is clearly given by the oscillograms 1, 2 and 3. The oscillograms 4 and 5, while not having a positive maximum near the axis, show also a striking tendency toward a maximum at this point, which is, however, masked by the rapid rise of the building-up curve in this part of the cycle.

Conclusion in Regard to the Electrolytic Detector.

1. The whole phenomenon of the rectification of small alternating currents by the electrolytic detector seems to be explicable in terms of the theory of electrolytic polarization.

2. The polarization capacity of the small platinum electrode is not entirely negligible, even with currents making only 60 cycles per second. The polarization capacity may, however, aid in producing rectified current as well as oppose this effect, and apart from the effect of this capacity on the tuning of the circuit need not detract from the utility of the rectifier as a detecter for electric waves.

3. The present conclusions in regard to the action of the detector is entirely in accord with Pupin's original brief description of the phenomenon as quoted above.

Comparison of the Electrolytic Detector with the Crystal Rectifiers.

The resemblance of the oscillograms with the electrolytic detector to those with the crystal rectifiers[1] is close, in so far as depends on the fact that both classes of rectifiers are nearly perfect[2] rectifiers when employed under their best conditions. The electrolytic rectifier in order to approximate perfection[3] as a rectifier must be

[1] Pierce, Part II., *l. c.*

[2] A rectifier is called "nearly perfect" when the ratio of the current in one direction to that in the opposite direction is large.

[3] The current through the electrolytic rectifier is slightly asymmetric when no polarizing current is employed.

polarized by the superposition of a direct current; while the use of the direct current with the crystal rectifier, though slightly lowering its resistance, does not materially improve the rectification. Also the two rectifiers are different, in that the electrolytic rectifier shows evidence of electrolytic polarization capacity, which, so far as may be judged from the oscillograms, is absent with the crystal rectifier. The experiment with the electrolytic detector, since it shows in the matter of polarization capacity the integrative action of this detector, which was sought for and not found with the crystal rectifier, is thus an interesting "control" experiment. Also an examination into the resemblances and differences of the two classes of rectifiers ought to aid us in seeking a rational explanation of the crystal rectifier.

However, before attempting to give definite expression to any theory as to the nature of the action in the crystal rectifiers, it is proposed to attempt to study in this connection the action of some form of vacuum-tube rectifier, so that the vacuum-tube rectifier may also be compared with the crystal rectifier under similar conditions.

In the meanwhile, the results of some experiments on a crystal rectifier making use of iron pyrites is presented in a separate paper to appear soon.

The expense of this research was in part defrayed from a grant from the Bache Fund of the National Academy, for which the author wishes to make grateful acknowledgment.

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April 14, 1909.

THE DEPENDENCE OF THE PHOTO-ELECTRIC CURRENT ON LIGHT INTENSITY.

By F. K. Richtmyer.

THERE seems to be a general tendency to assume that the rate of discharge of negative electrons from the surface of a metal under the action of light is directly proportional to the intensity of the incident light, at least within fairly wide limits. However, the available data on this subject seem rather contradictory. Elster and Geitel (1893),[1] working with cells made of potassium, show data that seems to prove that the two quantities are directly proportional for small ranges of light intensities. Lenard (1902)[2] worked with a very much greater range of intensities and reached the same conclusion, but his data when plotted do not give rigorously a straight line. On the contrary Griffith (1907)[3] working with ultra-violet light and a zinc plate concludes that the ratio of the current to the light intensity is not constant but increases with increasing illumination, and he gives reasons why this increase in the ratio ought to be expected.

Considering the present theory of the photo-electric discharge it is not quite clear why this proportionality should exist. The experiments of Lenard (1900, 1902, 1903),[4] Millikan and Winchester (1907)[5], and others seem to show that the emission of negative electrons is due to sort of an atomic disintegration. The complex atoms, becoming unstable under the action of light vibrations, eject electrons whose velocities do not differ greatly from the velocities which they possessed in the atom. Ladenburg and Markau (1908)[6] have shown that only those electrons are emitted whose natural periods correspond to the period of vibration of the incident light.

[1] Ann. der Ph., 48, 625, 1893.
[2] Ann. der Ph., 8, 149, 1902.
[3] Phil. Mag., p. 297, 1907.
[4] Ann. d. Ph.
[5] Phil. Mag., p. 188, 1907.
[6] Phys. Zs., p. 821, 1908.

But why should the number of such electrons emitted be strictly proportional to the light intensity? The following experiments were undertaken in order to get some data in addition to that given in the references cited above.

The apparatus is shown in Fig. 1. A 6-volt storage battery is connected in series with two resistance boxes R_1 and R_2, whose sum

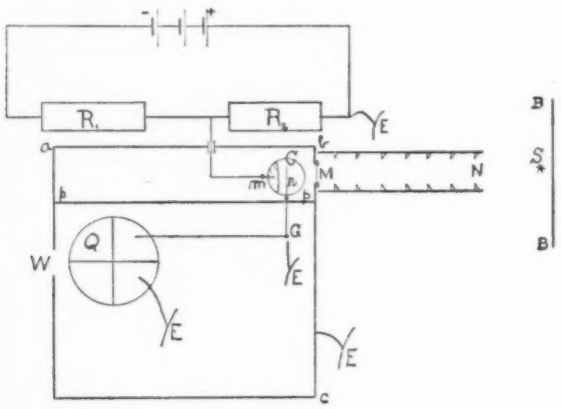

Fig. 1.

is kept constant, while either may be varied at pleasure, thus keeping a constant current in the circuit. The positive terminal of R_2 is connected to earth and the negative terminal to the Na cell, C.[1] The receiving wire r, of platinum, is connected to one pair of quadrants of the Dolezalec electrometer Q, the other pair being earthed. The electrometer was charged to + 55 volts by contact with the laboratory circuit and did not leak noticeably for several hours. The cell and the electrometer with all their connecting wires were encased in a tin box *abcd*, painted dull black on the inside. A window at W admitted light to the electrometer mirror, the cell being protected from stray light from this source by a partition *pp*. At G provision was made for making or breaking ground connection.

The cell was illuminated through the window M by a 16 c. p. lamp at S, a wooden box MN (square cross-section) protecting the cell from the stray light of the room. This box was at first simply

[1] See Elster and Geitel, Ann. d. Ph., 43, 225, 1891.

blackened on the inside, but serious error was caused by reflection from its sides. This difficulty was avoided by gluing laterally to the inside of the box small right-angled strips of wood as shown. Some annoyance was also caused by the irregularities in the transmission of light through the walls of the glass, due to varying angles of incidence as the position of S was varied. Also the distance between the plane of the filament of the lamp and the rather irregular surface of the metal was not easy to measure. The window M was therefore covered by a piece of medium weight writing paper. The intensity of illumination on this paper could be measured very accurately and the light transmitted to the cell was strictly proportional to the light intensity on the paper. Finally the source S was backed up by a black screen BB. Under these conditions a very great range of intensities was available by varying the distance MS and the number of layers of paper covering the window.

It may be of value to record some of the numerous difficulties that were encountered during the process of this investigation. It

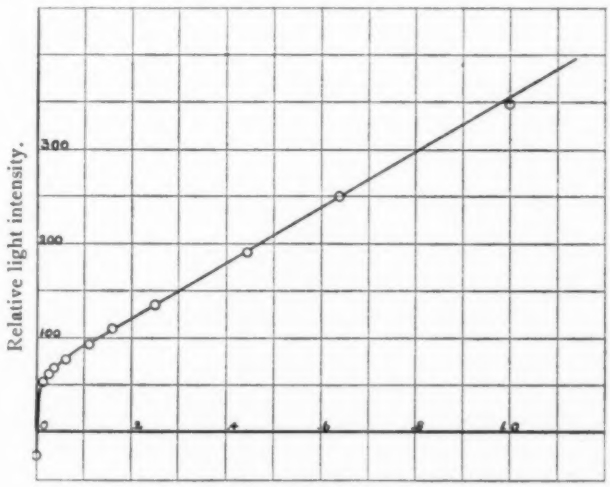

Photo-electric currents.

Fig. 2.

was noted that if the cell C was kept in the dark, the Na insulated (except for the high resistance ground through the walls of the

cell) and the ground connection at G broken, the electrometer showed a small rate of deflection in a direction to indicate that the needle was receiving a positive charge. If, however, the cell be illuminated by a gas flame 15 or 20 feet distant a small rate of deflection in the opposite direction was observed. And if the intensity of illumination was then gradually increased a curve of the form shown in Fig. 2, data Table I, resulted. The rate of charge was observed by a stop watch. (For these data the paper window at M was removed to get greater intensity.)

TABLE I.

Flame to Na Surface.	Light Intensity.	Time to Charge 100 Divisions.	Relative Currents.
1.00 ft.	1	28.8 sec.	.0347
1.25	.64	40.0	250
1.50	.445	52.6	190
2.00	.25	74.0	135
2.50	.16	91.0	110
3.00	.111	107	93
4.00	.0625	129	77
5.00	.040	149	67
6.00	.028	161	62
8.00	.0156	197	51
inf.	.000	400[1]	−25

The reason for the non-linear character of this curve is apparently connected with the reason for the positive charge being given to the electrometer when the cell is in the dark. This latter is probably due to the fact that the Na surface is highly electro-positive with respect to the platinum wire r. When G is grounded, r is at zero potential and the Na is at a positive potential equal to the contact potential difference between Na and Pt, the resistance between the two being the glass walls of the cell which are probably made conducting by a very thin film of Na, deposited by sublimation at the time of filling the cell.

If this explanation is correct, by charging m to a negative[2]

[1] Direction of drift opposite to that of the preceding.

[2] It is to be remembered that when we charge m to a negative potential, we really charge the platinum wire sealed in through the glass to that negative potential, whereas the Na itself is at a potential higher than this by an amount representing the contact potential difference between Na and Pt.

potential we ought to decrease this current or possibly reverse it. In fact we may consider the connections as represented diagrammatically in Fig. 3. The cell C tends to send positive electricity in a counter-clockwise direction, while the "battery" at e (the pd. around R_2, Fig. 1) tends to send current in the opposite direction, through the high resistance of the cell. The direction of the current will depend on which E.M.F. is the greater. We ought to expect a linear relation between this current and e.

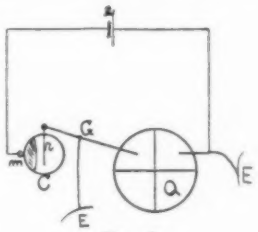

Fig. 3.

To test this the value of e was varied and the corresponding rate of charge of the electrometer measured by a chronograph. In taking these observations it was noted that the rate of deflection decreased as the electrometer became more highly charged, due to the fact that as the wire r became positive the effective E.M.F. of C was decreased. To avoid this disturbance due to the electrometer itself becoming charged, the following method of taking data was employed: Chronograph record was made of the transit of the spot of light on the electrometer scale past the 10, 20, 30, 40, etc., mm. mark. From this data a curve was plotted with rate of charge as ordinates and the position of the spot of light on the scale as abscissæ. This curve was then projected backward until it cut the ordinate representing the electrometer zero. See Fig. 4 and Table II.

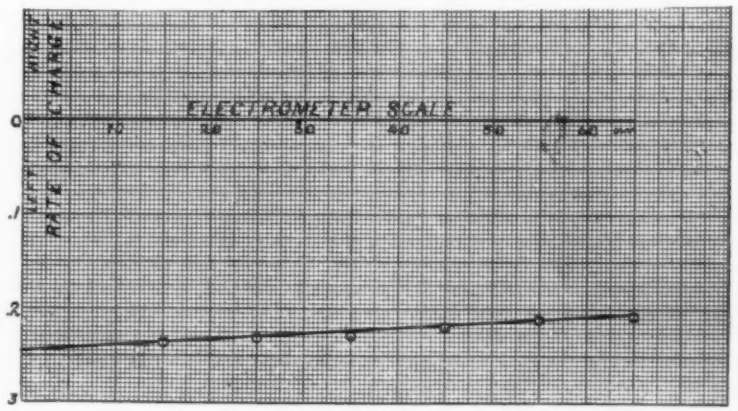

Fig. 4.

TABLE II.

For $e = 0$. Between Divisions	Requires	Rate of Charge.
10–20 mm.	42.1 sec.	.237 mm./sec.
20–30	43.2	.232
30–40	43.6	.229
40–50	45.4	.220
50–60	47.1	.212
60–70	48.1	.208

Rate of charge at electrometer zero = .248 mm./sec. as read from intersection on curve.

A series of such curves was obtained, one for each value of "e" used and their intersections with the ordinate for the electrometer zero were taken as comparative values of the current. A summary of this data is shown in Table III. and graphically in Fig. 5, 1.

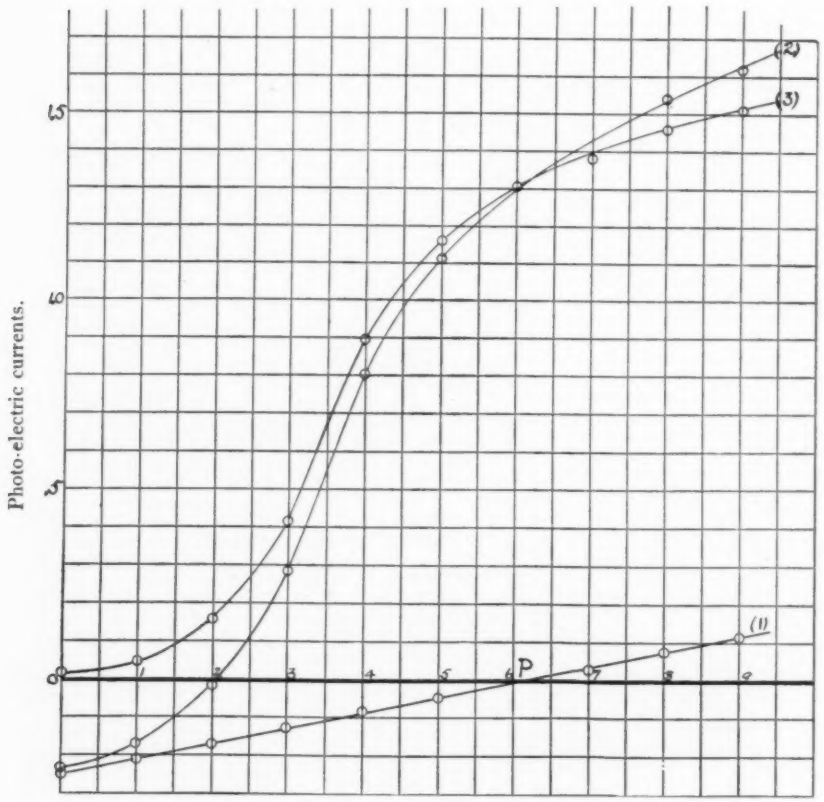

Fig. 5.

TABLE III.

Values of e (Arbitrary).	Direction of Charge.	Rate of Charge at Elec. Zero.
0	left	.248 mm./sec.
1	left	.217
2	left	.172
3	left	.126
4	left	.080
5	left	.044
6	too small to measure	
7	right	.039
8	right	.080
9	right	.117

(These arbitrary units of potential are nearly equivalent to .5 volt.) Since Fig. 5, 1 is a straight line the above explanation as to the origin of the current in the cell when not illuminated seems to be verified.

To compare the magnitude of this current due in part to contact pd. with that produced under the action of light a second curve was obtained in exactly the same manner as 5, 1 except that the cell was illuminated by a 16 c. p. lamp several feet away shining on the paper window *M*. This curve is shown in Fig. 5, 2. The effect of the light alone is clearly the difference between the ordinates of these two curves and is shown in Fig. 5, 3.

Now as is explained above, the potential *P* (Fig. 5, 1) represents the contact pd. between Na and Pt (about 3 volts). And when the platinum wire at *m* leading into the Na is charged to this negative potential the Na itself is really at zero potential. To represent properly the relation between the potential of the Na and the photo-electric current from it, Fig. 5, 3 should be replotted with the origin shifted to point *P*. This is done in Fig. 6. It may be noted that this curve agrees in the essential details with those obtained by Ladenburg and Markau[1] for the photo-electric current from platinum under the action of ultra-violet light.

Referring to Fig. 5 it is evident that the currents indicated by curve I. are large enough to cause serious error unless suitable corrections are made. This would not only be undesirable but would tend to lessen the accuracy of observations, since the correction is a fairly large percentage of the whole effect. These corrections

[1] *Loc. cit.*

would however be reduced to zero if during the investigation the cell were kept charged to a potential (about 3 volts) represented by point *P*. Whatever current is now obtained must be due to light alone. Under these conditions the variation of the photo-electric

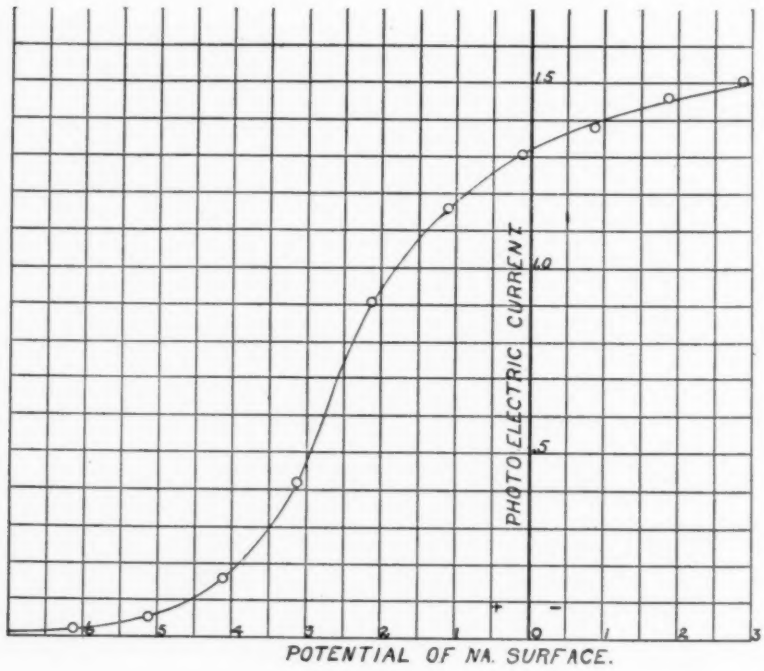

Fig. 6.

current with variation in light intensity could be quite accurately studied.

The measurement of current by use of the chronograph had several disadvantages. It took a long time to "read" the chronograph records. Besides, as the currents to be measured became larger the rate of drift of the electrometer increased proportionally. This caused certain irregularities due apparently to an increasing lag of the needle. This method was consequently abandoned and the current was measured by observing the permanent deflection given to the needle after being illuminated for a measured time interval, say 30 or 40 seconds. This time interval could be quite accurately obtained by switching on the lamp *S* simultaneously with

a beat of a 2-second sounder, counting the desired number of seconds, and breaking the circuit at another beat, after which the electrometer would come to rest and the deflection could be read. As the light intensity increased the duration of illumination could be correspondingly lessened so as to give approximately the same

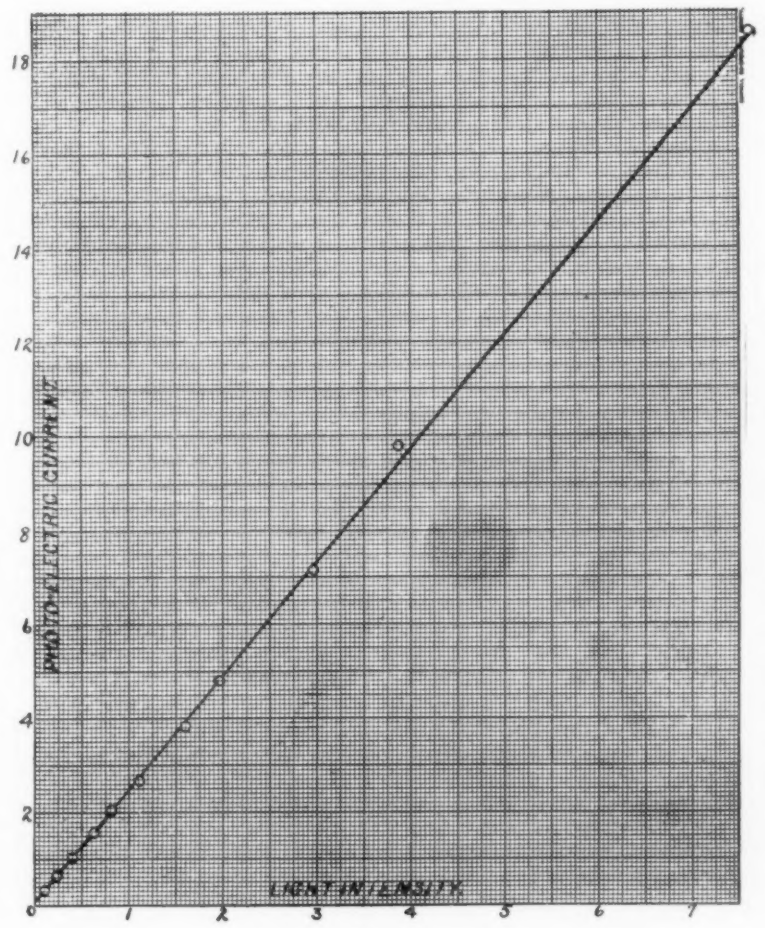

Fig. 7.

deflection in each case. From this data the deflection of the electrometer, had the duration of the illumination been only one second, could be readily computed, and thus the currents uniformly compared. The currents so obtained are the average of from 6 to 10 observations for each light intensity.

The results are shown in Fig. 7, data Table III. The first nine readings were made by allowing the light from the 16 c.p. lamp to fall on *two* thicknesses of paper at the window *M*, the intensity of illumination on the cell being estimated at .007 candle-feet for the first reading. Intensities and currents are given in relative values only. For the last three readings one sheet of paper was removed, its absorption coefficient being determined by observing the deflections before and after removing.

TABLE III.

Lamp to Window.	Relative Light Intensity.	Deflection per Second.
10.00 ft.	.0100	.318 mm.
6.50	.0236	.646
5.00	.0400	1.033
4.00	.0625	1.573
3.50	.0816	2.055
3.00	.111	2.682
2.75	.131	3.20
2.50	.160	3.93
2.25	.197	4.80
One paper removed, increasing illumination 4.75 times.		
4.00	.297	7.51
3.50	.387	9.79
2.50	.760	18.53

CONCLUSION.

The photo-electric current from a Na surface at zero potential, under the action of light from an incandescent lamp is strictly proportional to the light intensity for ranges of intensities from (approximately) .007 c.f. to .5 c.f.

It is hoped to extend this investigation to much higher intensities by use of improved apparatus. It is with pleasure that the writer acknowledges his obligations to Professor Merritt for his many suggestions.

CORNELL UNIVERSITY,
April, 1909.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

THE CRITICAL DENSITY OF WATER.[1]

BY HARVEY N. DAVIS.

IN 1897, Thiesen suggested that the empirical formula

$$L = L_0(t_c - t)^n$$

for the heat of evaporation of a substance would give, at the critical temperature t_c, the right value not only of L itself but also of dL/dt, if n is less than 1. His value, $n = \frac{1}{3}$, was based on the observations of Regnault, Dieterici and Griffiths. In 1906, Henning deduced from his own newly published observations, with those of Dieterici, Smith, Griffiths and Joly, the values $L_0 = 94.21$ calories and $n = 0.31249$, using $t_c = 365°$ C. A still better set of constants is $L_0 = 92.93$ mean calories and $n = 0.3150$. With these constants, Thiesen's formula represents not only the seventeen experimental values below 100° reported by the five investigators just mentioned, but also the eighteen means between 100° and 200° presented to this society by the writer a year ago, all with an average deviation of only one sixteenth of one per cent. The formula can, therefore, be used with some confidence for qualitative extrapolations above 300°.

The relation between the pressure and the temperature of saturated steam has been determined up to the critical point by several observers. The values of Cailletet and Colardeau will be used in this paper. They enable one to compute the dp/dt of Clapyron's equation and thus to get from the values of L contained above a set of values for the change of volume during vaporization at temperatures above 200°. But the 1905 edition of Landolt and Bornstein's tables gives values of the density of water up to 320°. It is therefore easy to compute a set of values for the density of saturated steam up to that temperature, and to verify, as in the accompanying figure, Cailletet and Mathias's "law of the straight diameter." The diameter turns out to be, as usual, nearly but not quite straight. Its points above 200°[2] are represented well within their limits of error by the equation

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 26 and 27, 1909.

[2] An attempt was made to include also the points below 200° but even a third degree ormula fails to represent satisfactorily the values near 0°.

$s = 0.4552 - 0.0004757\,(t - 160) - 0.000000685\,(t - 160)^2$ gr./cm.^{3}

The substitution of $t_c = 365°$ in this formula gives as the critical density of water

$$s_c = 1/v_c = 0.329 \text{ grams per cu. cm.},$$

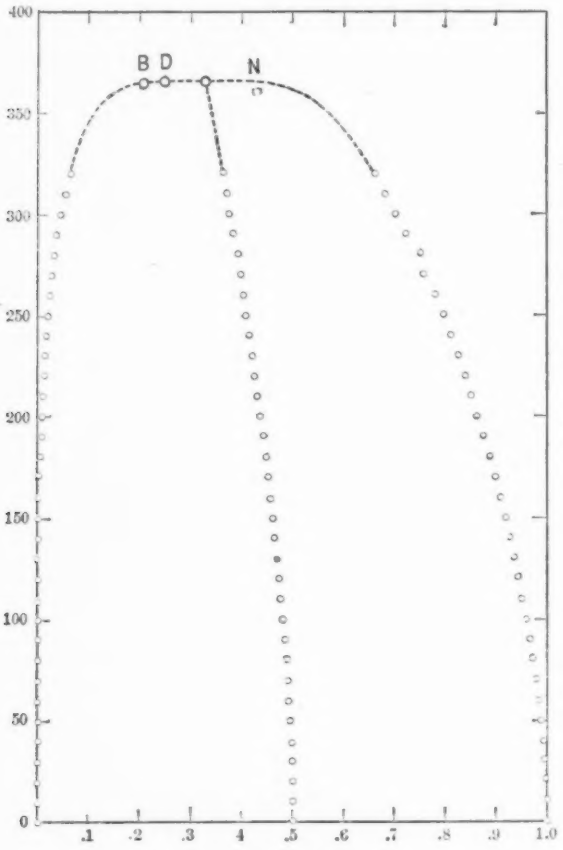

Fig. 1.

from which

$$v_c = 3.04 \text{ cu. cm. per gram.}$$

The accuracy of this determination is difficult to estimate because of its dependence on the extrapolation formula for L. It should, however, be noticed that comparatively large percentage errors in the computed densities of saturated steam make very small percentage errors in the mean densities and therefore in the critical volume. It should also be noticed that a considerable error in the assumed critical temperature makes only a small error in the critical volume.

In 1885, Nadejdime reported $v_c = 2.33$ cm.^{3}/gr., and in 1890, Battelli found $v_c = 4.812$ cm.^{3}/gr. Both used methods which were primarily designed for the determination of critical temperatures, and could be expected to give critical volumes only with considerable uncertainty. In 1904, Dieterici computed $v_c = 4.025$ cm.^{3}/gr. from an empirical formula of Young, assuming that at the critical point water is "normal." The disagreement between the present value and his indicates a polymerization factor of 1.3, which agrees very well with the first set of numbers given by Ramsay and Shields for water at ordinary temperatures, but cannot easily be reconciled with Ramsay's later corrected values.

From the equation of the diameter and the previously computed change of volume during vaporization one can compute the volumes and densities of water and of steam at temperatures above 320°. The results are indicated by the dotted line in the figure. The flatness of the top of the steam dome is due to the crowding of a large pressure range into a small temperature range.

JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

DISTRIBUTION OF GASES IN THE ATMOSPHERE.[1]

BY W. J. HUMPHREYS.

THE distribution of the principal gases of the atmosphere is calculated according to Ferrel's formula for places above the plane of the upper inversion. Below this plane, because of convection, it is assumed to be constant in volume per cent. except as modified by the presence of water vapor.

It is a striking fact that water vapor, which is so important in the economy of nature, often forms a smaller percentage of the atmosphere, even at the surface of the earth, than does argon; and that for the entire atmosphere it is seldom more than one fourth as great as is argon.

THE EXPONENT OF GAS RADIATION.[1]

BY W. J. HUMPHREYS.

BY taking into account the fact that, for earth radiation, water vapor is practically a black body, it is possible with a knowledge of its effective temperature, that is, the temperature of an equally radiating black body, and with a knowledge of the temperature of the isothermal region, to determine approximately the total-radiation exponent of the upper air.

In this way it can be shown that the total radiation of the dry air is nearly proportional to the fourth power of its absolute temperature, at least when its temperature is not far from that of the earth.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 26 and 27, 1909.

METHODS AND ERRORS IN MAGNETIC MEASUREMENTS.[1]

BY CHARLES W. BURROWS.

AFTER a brief introduction in which the merits of the common types of apparatus are discussed, the compensated double yoke method used at the Bureau of Standards is described. This is followed by a discussion of experimental data, showing the nature and magnitude of the various disturbing influences which enter into the measurement of normal magnetic induction and hysteresis data. These disturbing influences include galvanometer corrections; non-uniformities in specimens, magnetizing coils, and magnetic flux distribution; effect of ends of solenoids, of compensating coils, and of yokes; influence of imperfect demagnetization and of repeated reversals of the magnetizing current; effect of mechanical vibrations and temperature.

A comparison of data obtained by different methods is given, and the desirability of having magnetic data presented in round numbers and with some uniformity is shown. It is suggested that the magnetizing forces corresponding to the even thousands of gausses of induction would give satisfactory and easily comparable data.

In conclusion, specifications of the conditions and manner of measuring induction and hysteresis data, and an estimate of the limits of accuracy attainable, are given.

ACHROMATISM OF INTERFERENCE.[1]

BY C. W. CHAMBERLAIN.

THE production of a large number of interference bands from originally white light, in channelled spectra for example, is not a proof of the regularity of white light but of the resolving power of the spectroscope. The phenomena of white light may be produced by a succession of entirely irregular impulses. The production of a spectrum by means of a grating or prism involves the spreading out of an impulse over a length of time which increases with the resolving power. Interference in a spectrum involves a retardation of one portion of the impulse over the remainder. If an interfering system is illuminated with white light and the interfering beams fall upon a grating, the two portions of each white light impulse are spread out into a series of periodic impulses the number of which depends upon the number of lines in the grating. The relative retardation of the two portions of each impulse depends upon the angle between the lines of the grating and the interference fringes and the distance between the interfering system and the grating.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 26 and 27, 1909.

Heretofore in the use of the interferometer the central fringe alone has been made achromatic in white light, and only a very narrow region on either side of this fringe shows any periodicity whatever. The number of fringes seen in white light can be enormously increased by viewing the region on either side of the central fringe through a grating, the lines of which are perpendicular to the fringes, rotating the grating through ninety degrees, then increasing its distance from the dividing surface of the instrument as the distance from the central fringe increases. In this manner black fringes in white light can be observed with large difference of path.

The Absence of Photoelectric Fatigue in a very High Vacuum.[1]

By R. A. Millikan and G. Winchester.

In view of conflicting testimony as to the existence of photoelectric fatigue in a very high vacuum an extended series of observations was made upon silver, zinc, iron, nickel, and copper. In no case was any positive evidence of photoelectric fatigue obtained. On the contrary, continuous illumination by a very powerful arc largely increased the photoelectric current from all of the metals experimented upon. In each case, however, a maximum was reached which remained constant, within the limits of experimental error, even under the most prolonged illumination. Also after this maximum had once been attained it remained altogether invariable with time during the period during which tests were carried out — a period amounting, in the cases of silver and copper to more than three weeks.

The initial increase in the photoelectric current produced by powerful illumination can be accounted for by assuming either that the light produces a mechanical roughening of the surface and hence an increase in the effective illuminated area, or that the light cleans the surface of oxides, adherent or occluded gases, or other surface films. The absence of fatigue under powerful and prolonged illumination lends support to the view that the emission of electrons by metals under the influence of ultra-violet light is a specific property of the atom of each metal rather than a property imposed upon the metal by the occlusion of a gas. It further shows that the phenomenon of photoelectric fatigue as ordinarily observed is one which has its seat in the gaseous layer surrounding the metal rather than in the metal itself.

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 26 and 27, 1909.

THE KINETIC ENERGY OF THE POSITIVE IONS EMITTED FROM HOT BODIES.[1]

BY F. C. BROWN.

THESE determinations of the kinetic energy of the positive ions were carried out under essentially the same conditions as were those on the kinetic energy of the positive ions from platinum.[2]

The ultimate object of the experiment was not so much to measure the kinetic energy of the ions as it was to test the theory proposed in a paper by Richardson and Brown.[3] According to this theory the kinetic energy of the ions arises from thermal agitation. This requires that the current carried from a portion of an infinite plane to a neighboring parallel plane should be expressed by the formula,

$$i = i_0 e^{-\frac{nev}{R\theta}}$$

where R is the ordinary gas constant in the equation $R\theta = \frac{1}{3} nm\bar{v}^2$.

The summary of the result is given in the accompanying table. Within the limits of experimental error these results are in agreement with the theory.

	Form of Radiator.	Temperature, Absolute.	Current at Zero Potential, $i_0 \times 10^{12}$.	Pressure, mm.	Gas Constant, $R \times 10^{-7}$.
Gold (1)	Disc	1030 973	1	.007	4.2
Gold (2)	Disc	1190 1163	60	.01	3.9
Silver (1)	Disc	1020	.8	.002	3.0
Silver (2)	Disc	1150	35	.008	2.9
Palladium	Disc	1170	25	.04	3.4
Aluminium phosphate	Disc	1230	100	—	3.9
Aluminium phosphate (2)	Disc	1170	120	.006	3.4
Nickel	Strip	1120	2.5	.003	3.6
Iron (1)	Disc	1100	1	—	4.6
Iron (2)	Disc	1100	.8	.005	5.2
Iron (3)	Disc	1240	—	.01	4.4
Platinum	Wire	1695	—	—	5.1
Tungsten	Filament	1150	1	.0003	5.1
Tantalum	Filament	1050	.7	.0005	9.6
Tantalum	Strip	1050	1.0	.002	3.0
Osmium	Filament	1120	3.0	—	2.5

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 26 and 27, 1909.

[2] Paper by F. C. Brown, Phil. Mag., 36, vol. 17, p. 355; PHYS. REV., XXVII., 528.

[3] Phil. Mag., S. 6, vol. 16, p. 353, 1908.

A Spectroscope of High Resolving Power.[1]

By C. W. Chamberlain.

LIGHT falls upon a concave or plane grating having previously been separated by successive reflections into a succession of impulses separated from one another a distance in wave-lengths approximately equal numerically to the number of lines in the grating. A preliminary report. The theory has been developed and will be published later. The experimental work is now in progress.

Account of the Treasurer of the American Physical Society for the Year ending December 31, 1908.

Disbursements.

Expenses of Secretary, 1907	$106.70
Expenses of Treasurer, 1907	30.45
Allowance to Secretary, 1908	100.00
Mileage, etc., of Secretary, 1908	73.88
Allowance to Treasurer, 1908	50.00
Stationery and Printing, orders of Secretary	147.64
Stationery and Printing, orders of Treasurer	92.95
Stationery and Printing, account of A.A.A.S.	5.68
Extra expenses in connection with transfer of office of Treasurer	35.95
Repayments to members owing to errors in 1907	18 00
Physical Review for 1908 and in part for 1907	1,744.00
("Science Abstracts") for 1908 and in part for 1907	1,359.25
Deposit in Bank, December 31, 1908	114.44
	$3,878.94

Receipts.

William Hallock, Treasurer	$1,020.00
Dues for 1906 and before	30.00
Dues for 1907	457.50
Dues for 1908	2,224.78
Dues for 1909 and after	20.00
Life Membership	50.00
Entrance Fees	45.00
Payments for Section "B," Science Abstracts	13.00
Interest on Deposit	18.66
	$3,878.94

J. S. Ames, *Treasurer.*

I certify that I have audited these accounts and that I find them correct.

J. B. Whitehead.
February 11, 1909.

[1] An abstract of a paper presented at the Washington meeting of the Physical Society, April 26, 1906.

NEW BOOKS.

An Elementary Manual of Radiotelegraphy and Radiotelephony for Students and Operators. By J. A. FLEMING. 8vo, pp. xiv + 340. London, Longmans, Green & Co., 1908.

This manual is more than an abridgement of the author's "Principles of Electric Wave Telegraphy," still the standard reference book on the subject. The larger work has been intelligently and carefully condensed into the smaller volume, and many of the important results of the rapid development of the art since the publication of the earlier book in 1906 have been included. The manual includes, for instance, the most complete discussion of the Poulsen arc to be found anywhere in the literature up to the present, and a short account of the crystal rectifiers is added to the list of oscillation detectors treated in the older work. The chapter on Radiotelephony includes the important developments in this subject down to August, 1908.

Space has been gained by omitting much of the historical matter and the derivation of most of the formulas. The rapid progress of the last two years has also enabled the author to sift out many of the tentative devices and arrangements of apparatus which were included in the earlier work, and to concentrate his attention on those features which have proven most valuable in practice. But the absence of all references to more complete treatments of even the most important subjects seems unfortunate.

Some errors of the larger work have been corrected in the manual. In the chapter on Electromagnetic Waves, which is written with all the author's usual clearness and force despite its highly condensed form, the Maxwell relations between the time and displacement rates of change of the magnetic and electric forces are now given correctly for a plane wave, as

$$\frac{d^2H}{dt^2} = \frac{1}{\mu K}\frac{d^2H}{dz^2} \quad \text{and} \quad \frac{d^2E}{dt^2} = \frac{1}{\mu K}\frac{d^2E}{dz^2}$$

(page 125). Other errors have crept in, however, and Fig. 7, on page 126, which represents the electric and magnetic forces as sine and cosine waves, displaced 90°, is incorrect and misleading. The correct diagram for this case, showing the electric and magnetic waves in the same phase, is given in Maxwell's Electricity and Magnetism, Vol. II., p. 439, Fig. 67.

Besides its uses as a hand-book for operators and an introduction to the more advanced treatises for students, the manual will serve as a valuable supplement to the author's "Principles of Electric Wave Telegraphy" until a new edition of the larger work shall become necessary.

G. W. NASMYTH.